THE UNIVERSITY OF ALBERTA

THE EFFECTS OF INTERNATIONAL CAPITAL MOVEMENTS ON EMPLOYMENT UNDER A FLEXIBLE EXCHANGE RATE

by

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ABSTRACT

The question of what effects capital inflows have on the level of employment is of major importance for the Canadian Economy. Unfortunately, however, this problem has received very little attention from Canadian economists. This thesis is an attempt to show, by the examination of a theoretical model, the conditions under which capital inflows will increase or decrease the level of employment.

The assumptions of the model are Keynesian and the method is comparative statics. A two country world is assumed, and the real national income of these two countries, and the exchange rate are the three variables of the system. Expressions for the effects of changes in the capital inflow on these three variables are then derived, and the equation for the change in country 1's real income is examined in some detail.

The analysis leads to one major conclusion: that there are no a priori grounds for assuming that capital inflows will either decrease or increase the level of real national income and employment. The effects that capital inflows will have are found to depend on the way in which the capital is used, and on the values of the elasticities and propensities of the two countries. A number of sets

of values are assumed for the different parameters, in an attempt to gain some idea of the conditions which would be necessary to insure that a capital inflow would be expansionary. The results are of rather doubtful significance, however, because of the uncertainty that surrounds the values of the parameters. The thesis concludes on the note that further analysis must await the statistical estimation of the parameters: a task which is probably even more difficult than it is important, but one which is necessary if this very important question is to be resolved.

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CHAPTER I

INTRODUCTION AND BACKGROUND

1. The Object of the Thesis

The object of this thesis is to examine some theoretical relationships between capital[1] flows and the level of employment and national income, for a country which has a flexible exchange rate. The approach will be to set up a simple, three equation, two-country model, with the income of each country, and the exchange rate as variables. The model will then be analysed by the comparative statics method. The three equations will be differentiated with respect to a shift variable, and equations for the change in the income of country 1, the change in the income of country 2, and the change in the exchange rate will then be derived. The equation for the change in country 1's income will then be examined in order that it may be determined whether a capital inflow will be contractionary[2] or expansionary. This analysis will be facilitated by

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[1] Throughout this thesis the word "capital" will always refer to money capital.

[2] The word "contraction" will be used throughout to mean a decrease in employment or the level of national income, and conversely for "expansion". The relationship which is here implicitly assumed to exist between employment and real national income, and the difficulties involved in defining real national income, will be discussed in Chapter II.

classifying the capital flows into types according to end-use, and examining each of these types separately. It may, of course, be found that the direction of the change in national income will depend, not only on the type of capital flow, but also on the values of the parameters of the system. This being the case, an attempt will be made to calculate some of the borderline conditions for the general model; these borderline conditions, of course, being nothing more than sets of values of the parameters which will make the change in national income equal to zero. No attempt will be made, however, to estimate the values of the parameters for the Canadian, or any other economy, such a task being outside the scope of this thesis.

An attempt will also be made to show whether changes in some of the parameters have more effect on the level of national income than changes in others. To do this, arbitrary values of the parameters will be assumed, and then each will be changed by a constant percentage and the effect on national income will be calculated. This is deemed important because of the degree of uncertainty that surrounds actual estimates of the values of the parameters. If variations in some parameters are found to be much more influential than variations in others, then these are the ones for which accurate estimates would be most important.

The argument of this thesis, while arising from an episode in Canada, is mainly of a general theoretical nature. The major part of the analysis could be applied to

any country and only in the final sections is the model related specifically to Canada.

2. The Background

The idea for this study developed from what came to be known as the "Coyne Controversy". In late 1959 and throughout 1960 Mr. James E. Coyne, then the Governor of the Bank of Canada, made a number of speeches[3] throughout Canada, in which he placed most of the blame for the then high level of unemployment and the low rate of growth, on the high level of foreign, and particularly American, capital which had been entering Canada. All were not convinced, however, and in particular, a group of eastern economists were very disturbed by Mr. Coyne's remarks.[4] So disturbed were they, in fact, that they sent a letter to the Minister of Finance requesting that Mr. Coyne be removed from office. This unprecedented move, and the "leak" of the contents of the letter to the Canadian press caused a great deal of controversy throughout the country. The Conservative Government of the day, under the leadership of Prime Minister Diefenbaker, had also become displeased with

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[3] See particularly James E. Coyne, "Foreign Debt and Unemployment," remarks delivered at a meeting of the Canadian Club of Toronto, November 14th, 1960 (mimeographed by the Bank of Canada).

[4] Mr. Coyne's view on capital inflows was by no means the only issue between himself and "the economists". Their main criticism was that the monetary policy which Mr. Coyne had pursued since taking office as Governor of the Bank of Canada was completely inappropriate for the existing economic situation.

Mr. Coyne, and consequently, in June, 1961, introduced to the House a bill relieving Mr. Coyne of his duties as Governor of the Bank. This bill was quickly approved by the large Conservative majority of the House of Commons. The Senate,[5] however, refused to pass the bill, and called on Mr. Coyne to testify before a Senate Committee. The Senate exonerated Mr. Coyne of the charges which had been made against him, upon which he immediately resigned. A new Governor was subsequently appointed and the controversy was soon all but forgotten. The issues, however, and particularly the question of what effects capital inflows have, remained, and still remain, largely unsolved. While some literature did appear on the subject, much of it is very unacademic in nature, and even that which does exhibit some degree of economic subtlety leaves one rather unconvinced.

Thus we have the genesis of this thesis; a problem which, it appears, has not only not been solved, at least to the satisfaction of all, but which has also been largely ignored.[6] This thesis is an attempt to throw some

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[5] The Senate at this time was predominantly Liberal.

[6] This should not be interpreted to mean that the theoretical problems of capital flows under flexible exchange rates have been ignored by economists. While quite a good deal of literature exists on the subject in general, very little attention appears to have been given to the Canadian experience of the last twelve years. See, for example, Irving Brecher and S. S. Reisman, Canada-United States Economic Relations (Ottawa: Royal Commission on Canada's Economic Prospects, 1957). Referring to foreign investment in Canada, they say: " . . . it is worth restating the fact that this is an area of Canadian economic

further light on this very important Canadian problem.

The method adopted can be considered to be of three stages: the first, consisting of Chapter I, being the introduction and background, the second, consisting of Chapter II, being the assumptions of the model and the model itself, and the third, consisting of Chapters III and IV, being the implications of the model and the conclusions. The rest of this chapter, then, will be a brief review of the relevant literature with emphasis on that which specifically refers to the Canadian situation.

3. A Survey of the Literature

Since Mr. Coyne originated the controversy, it seems only proper that we begin with an analysis of his contribution. While a number of Mr. Coyne's statements are relevant, his most succinct statements were made in a speech delivered in Toronto on November 14th, 1960. His position is perhaps best illustrated by the following quotations:

> There are many contributory causes of the growing inbalance of the Canadian trade and payments and the development of chronic or structural unemployment. My own view is that excessive reliance on foreign capital was probably the most important active cause.[7]

A few pages later he goes on to say:

> For example, when a provincial government borrows U.S. dollars in New York, it cannot make use of the U.S. funds except by selling them for Canadian dollars, and the rate of exchange will move until someone else is

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activity in general, and of Canadian-United States relations particularly, in which there has been little systematic analysis." p. 154.

[7] Coyne, op. cit., p. 9.

> found who is induced, by the discount on the U.S. dollars (or the corresponding premium on Canadian dollars) to exchange his Canadian dollars for U.S. dollars and use the U.S. dollars to buy and pay for imports--imports which would not take place otherwise; and which cause unemployment in Canada, or prevent employment from rising.[8]

and later:

> A reduction in the inflow of capital must, therefore, at the very least, go hand in hand with action designed to improve the current account of the balance of payments and improve employment in useful production in Canada.[9]

Although Mr. Coyne's stand, as illustrated by the above quotations, seems to be quite clear, it should be pointed out that this does not appear to always have been his position. Early in 1960, he expressed the opinion that foreign borrowing was one of the causes of inflation.[10] We cannot be certain, from Mr. Coyne's statements, whether he felt that the one situation existed at one time, and the other situation existed at another, or alternatively, whether he felt that both could exist at the same time. It is clear, however, that Mr. Coyne felt that capital inflows were definitely harmful to the Canadian

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[8] Ibid., p. 11. [9] Ibid., p. 13.

[10] See, for example, James E. Coyne, "Living Within our Means," remarks delivered at a meeting of the Canadian Club of Winnipeg, Winnipeg, January 18, 1960 (mimeographed by the Bank of Canada). On page 22, Mr. Coyne says: " . . . willingness to do without foreign borrowing, willingness to hold spending programmes to amounts that can be raised at home out of revenues, or available loanable resources at home, will reduce inflationary pressures and the volume of imports . . . " In this same speech, however, (see page 18) Mr. Coyne indicates that long term capital flows have aggravated the structural unemployment problem. Possibly then Mr. Coyne did feel that capital inflows could be a cause of both inflation and unemployment.

economy. They caused inflation and/or unemployment, both of which were extremely undesirable in his view. For the purposes of our analysis, the important thing is clear; that Mr. Coyne felt that capital inflows caused unemployment. Since Mr. Coyne does not give us any indication of the reasoning he used to reach his conclusions, we can go no further in an analysis of his views.

Early in 1961, Professor Gordon published a short monograph in which he explained why the twenty-nine economists had asked the government for Mr. Coyne's dismissal. Professor Gordon is scathing in his criticism of Mr. Coyne. Of Mr. Coyne's speeches he has this to say:

> many of the arguments contained in them seem to be confused, and some are clearly misinformed or fallacious.[11]

Discussing Mr. Coyne's general competence, he says:

> The Governor's picture of some of the most fundamental economic processes is, to say the least, peculiar. He does not, for example, seem to understand the process by which national income is generated, nor does he seem to have a firm grasp of the mechanism by which a nation's balance of international payments is adjusted. His interpretations of current Canadian economic conditions and problems appear to suffer considerably from this lack of general competence in economic analysis as well as from lack of knowledge or consideration of important facts.[12]

Professor Gordon then goes on to discuss Mr. Coyne's views on foreign capital inflows. He is very critical of Mr. Coyne's reasoning, but unfortunately fails to make his

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[11] H. Scott Gordon, The Economists Versus the Bank of Canada (Toronto: The Ryerson Press, 1961), p. 37.

[12] Ibid.

own position on the issue clear. The closest he comes to expressing his own opinion is the following:

> If the funds we borrow abroad lead to such an expansion in our ability to produce that we can pay the foreign lenders their interest and dividends and still have a substantial part of this new production left over for ourselves, are Canadians the losers by the transaction? Are we "living beyond our means" when we use foreign capital to increase the output capacity of our economy so that both we and the foreign lenders are able to reap economic benefits? To describe the Canadian economy in the way Mr. Coyne has done is a curious use of words, to say the least, but it is surely something more: it is muddle-headed economics.[13]

Professor Gordon, then, seems to feel that capital inflows have not, in fact, been deflationary, but on the contrary, have "increased the output capacity of our economy". He further goes on to say that "unemployment has been far more costly to us in Canada then our foreign borrowing".[14] This seems to imply, although he does not explicitly say so, that he thinks unemployment and foreign borrowing are not directly related. While Professor Gordon's criticism of Mr. Coyne is very harsh, it is also, unfortunately, of a very negative nature. Mr. Coyne, while he may not always have been right, and while he seldom made his reasons and assumptions explicit, usually made a positive statement as to what he believed. We are unable to say even this much for Professor Gordon.[15]

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[13] Ibid., p. 43. [14] Ibid., p. 44.

[15] Reviewing Gordon's book in The Winnipeg Free Press, February 28, 1961, R. C. Bellan had the following to say, "The economists' case against Mr. Coyne is weak indeed if it is based solely on Professor Gordon's assumptions and evidence. This reviewer, previously undecided, was persuaded by the book that Mr. Coyne must be right."

Professor Barber has expressed views similar to Mr. Coyne's with regard to the effect of capital inflows on the Canadian economy.[16] He says, for example:

> Canada has been borrowing funds from other countries to finance the purchase of imports which directly or indirectly could have been produced with her own unemployed resources. Not only has this increased unemployment levels in Canada, it has also added unnecessarily to the size of our external debt.[17]

Professor Barber argues that since Canada has been a net importer of capital, it necessarily follows that the level of final domestic expenditure has been larger than Canadian output by the amount of this net inflow. A diversion of this "excess expenditure" from foreign goods to Canadian output would, then, have substantially reduced Canadian unemployment. In conclusion he says:

> I would say that Canada's high level of unemployment during the past few years has been primarily due to the government's failure to encourage a reduction in the capital inflow and in the size of the current account deficit.[18]

Professor Barber is assuming here that the capital inflow has reduced spending on Canadian produced goods. This could come about either by a reduction in exports or a reduction in domestic spending on Canadian produced goods, or, of course, by a combination of these. Exports will decrease, ceteris paribus, only if the capital inflow appreciates the Canadian dollar, and then only if

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[16] Clarence L. Barber, "Canada's Unemployment Problem," Canadian Journal of Economics and Political Science, XXVIII (february, 1962), 88-102.

[17] Ibid., p. 91. [18] Ibid., p. 101.

foreign elasticities of demand for our exports are such as to cause a decrease in the volume bought when prices rise (i.e. if foreign elasticities of demand for imports are greater than zero) and if there are no foreign trade repercussions which could increase demand for our exports. Reduction in Canadian absorption of Canadian-produced goods implies that the Canadian dollar will appreciate, that this appreciation will cause imports to rise, and that these imports will take the place of domestically-produced goods. While it could be argued that this is what we would normally expect, these results are by no means certain.

Professor Barber has also ignored two other important considerations. First of all, the appreciation of the currency is equivalent to a reduction in the prices of imports, which in turn implies a fall in the price level, the extent of the fall depending on the amount of appreciation and on the proportion of total expenditure made on imports. This reduction in the price level is, ceteris paribus, equivalent to an increase in real income, and if we assume the absence of a money illusion, which implies that our propensities to spend are based on real income, then real consumption will rise, which, of course, must increase real output. This effect is closely related to the increase in imports which the appreciation of the currency brings about, and, in fact, together they are the aggregate counterparts of the familiar income and substi-

tution effects. Since the total effect must be divided between these two, it would seem important to take account of their relative magnitudes when attempting to determine the nature of the change in national income.

Professor Barber also seems to have ignored the indirect effects which capital inflows could have. He has not, for example, considered the effect on employment that this capital could have if it were used to build new factories or develop new resources, thus creating new jobs for Canadians. While he may have assumed that the foreign capital simply replaced Canadian capital, there is evidence that this has not always been the case, for much of recent Canadian development, especially of resources, could not have been done by Canadians, either because of the size of the projects, or because of the fact that many Canadian resources have been developed to serve specific United States markets, markets which would not be available to independent Canadian companies.

Other indirect effects which may influence national income and employment are the "resource - reallocation"[19] effect, and the monetary effect. Relative prices within a country are very likely to change if the exchange rate changes, for the prices of import-competing goods will tend to follow import prices, and exporting firms may tend

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[19]See Fritz Machlup, "Relative Prices and Aggregate Spending in the Analysis of Devaluation," American Economic Review, XLV (June, 1955), p. 265.

to adjust the prices of the goods they sell on the domestic market, if the price of the goods they export changes. The resource - reallocation effect refers to the shift of resources towards the higher priced goods, brought about by these price changes. This reallocation of resources could improve the efficiency of the economy and consequently increase real output. Machlup feels that this could be one of the more important effects of a change in the exchange rate. There is, of course, no way of knowing whether the effect will be positive or negative, and whatever the change in output, we have no way of knowing how it will affect employment.

A number of authors have stressed the importance of the interest rate and the money supply on the level of employment and national income under a system of flexible exchange rates.[20] It is sufficient to say, perhaps, that a discussion of the effects of a capital inflow on employment would be incomplete without some consideration being given to monetary factors. Admittedly, Professor Barber did suggest that proper monetary policy could have reduced unemployment with the existing level of capital inflow. Our concern here, however, is not how unemployment can be

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[20] See particularly S. C. Tsiang, "The Role of Money in Trade-Balance Stability: Synthesis of the Elasticity and Absorption Approaches," American Economic Review, LI (December, 1961), 912-935; and Robert A. Mundell, "The Monetary Dynamics of International Adjustment Under Fixed and Flexible Exchange Rates," Quarterly Journal of Economics, LXXIV (May, 1960), 227-257.

reduced, but rather what effect capital inflows via monetary factors have on employment. This aspect Professor Barber seems to have ignored.

There are, of course, many other factors which have to be considered when trying to determine the net effect of capital inflows. Among these are the effects of short term capital movements, the use to which the capital is put, and the effects on the investment and saving plans of Canadians. It would be impossible, of course, to consider all these factors either mathematically or verbally and hope to come up with a definite answer. Professor Barber's mistake was not so much that he failed to take all these things into account, but that apparently, without taking any of them into account, he reached a definite policy conclusion.

Professor Penner has discussed the probable effects of long term capital inflow on the Canadian economy, and has taken many of these factors into account.[21] He begins by saying:

> . . . in this paper I hope to show, first, that there is no theoretical reason to believe that the type of long-term capital inflow entering Canada during the 1950's is necessarily deflationary on balance; . . . and that while the empirical evidence is far from clear, and not too reliable, it does not favour the hypothesis that the capital inflow has in fact been deflationary.[22]

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[21]Rudolph G. Penner, "The Inflow of Long-Term Capital and the Canadian Business Cycle 1950-1960," Canadian Journal of Economics and Political Science, XXVIII (November, 1962), 527-542.

[22]Ibid., p. 528.

Penner's approach is to classify long-term capital inflows according to the way in which they are spent. He distinguishes three major types. "First, the foreign loan may be used to finance purchases of newly produced goods and services in Canada. Second, foreign borrowing may be undertaken for the express purpose of financing imports of foreign goods. Third, the capital inflow may be used to finance purchases of already existing real or financial Canadian assets."[23] He next assumes that the borrowing country is in internal equilibrium and that before the capital inflow, exports equal imports. He also assumes temporarily that there are no speculative or accommodating short-term capital flows and that there is no official intervention in the foreign exchange market. Professor Penner then attempts to determine the initial impact of each of the three above mentioned types of capital inflow.

For capital inflows of type one, he argues that since all the money will be spent in Canada, it must be converted into Canadian dollars by bidding up the price of the Canadian dollar until an import surplus equal to the capital flow develops. Then, "if we assume that Canadian financed expenditures on foreign plus domestic goods remains constant",[24] there will be a reduction in the demand for Canadian goods equal to the import surplus. Thus, when the capital inflow is used to buy Canadian goods it exactly offsets the reduction in demand due to the import

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[23] Ibid. [24] Ibid., p. 529.

surplus and aggregate output is not affected. He then goes on to discuss the indirect effects of the capital inflow on the savings and investment plans of Canadians, and he concludes, that while it is impossible to give any definite answer, he suspects that on balance this type of capital flow has been expansionary.

While Penner's analysis of the indirect effects of the capital inflow appears to be sound, his analysis of the direct effect deserves further attention. As he admits, the crucial assumption in his analysis is that total Canadian-financed demand for foreign plus domestic goods remains constant. There are two ways in which this assumption can be attacked. First of all, the real income effect mentioned earlier in conjunction with Professor Barber's article has been ignored. When the price level falls due to the appreciation of the currency, real consumption and real income must rise if money expenditure is constant. If real expenditure is constant when the price level falls, the same physical quantity of goods can be bought with less money, and thus money expenditure is reduced. Thus either in real or money terms, expenditure must change when there is an appreciation of the exchange rate.

The above assumption can also be attacked on the grounds that there is no <u>a priori</u> reason for assuming that every increase in spending on imports will be matched by an equal decrease in spending on domestically produced

goods. In fact, such a relationship seems highly unlikely. It is certainly conceivable that a reduction in the price of certain foreign goods could cause Canadians to spend money which would otherwise not have been spent. Such spending could be financed from idle balances, from a reduction in saving, or simply by borrowing from the commercial banking system. This increase in spending will be most evident if there is a low degree of substitutability between imports and domestic goods. It is even conceivable that a decrease in the price of imports could cause domestic spending on Canadian produced goods to decrease more than imports rise.

The price elasticity of demand for imports is another factor which must be taken into account in trying to establish the effect on national income of a change in the exchange rate. In fact, it is shown in Chapter III that, under the assumptions of our model, the elasticity of demand for imports, along with the ratio of the change in domestic consumption of domestically-produced goods to the change in imports, are all that is required to establish the sign of the change in national income due to a change in the exchange rate.

Penner's argument that national income will not be affected by capital inflow if all the money is spent on domestically produced goods is thus rather unconvincing, and, in fact, it is shown in Chapter III that his assumption that the increase in imports will be matched by an

equal decrease in Canadian expenditure on domestically-produced goods, can hold only if the elasticity of demand for imports is infinite.

Penner next goes on to discuss the effects of a capital inflow which is used to finance purchases of foreign goods. He concludes that since such a transaction does not go through the foreign exchange market, it will not affect relative prices and thus will not directly affect the level of national income. When indirect effects are taken into account, however, no definite conclusion can be reached.

He then discusses his third type of capital inflow; that used to finance purchases of already existing real or financial assets. He argues that since this type of capital inflow will have the same effect on the exchange rate as inflows of type one without creating demand for domestically-produced goods, this type of capital inflow is more likely to be deflationary than the other two. As before, however, the indirect effects may confuse the situation and thus no definite conclusions are possible.

Penner then goes on to discuss the effects that short term capital flows may have had, and his findings indicate

> . . . that the long-term capital inflow may have provoked short-term capital outflows and transactions by the Exchange Fund Account that have increased the probability of a positive relationship between the long-term capital inflow and Canadian National Income; . . .[25]

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[25] Ibid., p. 528.

Penner concludes his paper by saying that:

> . . . the theoretical and empirical evidence presented in this paper does seem to indicate that the type of capital inflow entering Canada during the 1950's certainly was not highly deflationary; the strong possibility exists that it was in fact expansionary on balance.[26]

While this concludes the review of the literature on the effects that capital flows have had on the Canadian economy, quite a good deal has been written on the general question of flexible exchange rates and capital flows, some of which is deserving of brief mention.

The "transfer problem" has always received considerable attention from economists, and the literature on the subject dates back to Mill and Ricardo. The classical economists, however, thought in terms of the gold standard and full employment, so their contributions are not particularly relevant to this discussion. Even since the gold standard was abandoned and full employment ceased to be thought of as a necessary long-run condition, most of the literature has assumed a fixed exchange rate, and thus is not directly applicable. The wealth of literature on the effects of a devaluation throws some light on the question, but even in this area conclusions which hold for fixed exchange rates may be inappropriate for a system of flexible rates. In recent years, however, and especially since the Second World War, flexible exchange rates have been growing in favour with economists, and consequently

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[26] Ibid., p. 542.

quite a good deal of literature has emerged, although much of it does not deal intensively with the problem of capital flows.

Most of the writers who have dealt with the transfers of funds under flexible exchange rates have assumed that they are on balance deflationary, but unfortunately very little time has been spent in trying to justify this assumption. Wonnacott, for example, in his book, The Canadian Dollar, 1948-1958,[27] while he does not make any explicit statement as to the effects of capital inflows on the level of national income, assumes a deflationary effect when he says, " . . . unemployment, which would be aggravated by an appreciation of the currency on the international exchanges under flexible rates."[28] R. E. Caves, in discussing the recent Canadian experience with flexible rates says, "As one would expect, the capital inflow increased, the Canadian dollar tended to appreciate, and the unemployment problem grew all the worse."[29] R. A. Mundell has recently expressed the same sentiment: " . . . an increase in foreign demand . . . would cause an appreciation of the exchange rate and therefore increased

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[27] Paul Wonnacott, The Canadian Dollar, 1948-1958 (Toronto: University of Toronto Press, 1960).

[28] Ibid., p. 12.

[29] Richard E. Caves, "Flexible Exchange Rates," American Economic Review, Papers and Proceedings, LIII (May, 1963), p. 128.

unemployment."[30] Other authors have expressed much the same idea[31] but few have attempted to prove it rigorously. A notable exception is E. Sohmen, who, in his recent book, Flexible Exchange Rates,[32] attempts to show that an appreciation of the exchange rate will lower national income and employment. Sohmen's analysis is open to attack on a number of grounds, however, and his conclusion that:

> Strictly Keynesian assumptions leave us, indeed, with the result that under flexible exchanges and in the absence of capital transfers a boom in one country will lower national income and employment in the rest of the world.[33]

is not altogether convincing. Sohmen's arguments and the criticisms of these arguments are presented in Appendix A.

While the literature so far reviewed has mainly expressed the view that capital inflows are deflationary under flexible exchange rates, this is not universally accepted. J. B. Williams, for example, has expressed the opinion that capital transfers can be either expansionary

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[30] Robert A. Mundell, "A Theory of Optimum Currency Areas," American Economic Review, LI (September, 1961), p. 664.

[31] See, for example, J. L. Stien, "The Optimum Foreign Exchange Market," American Economic Review, LIII (June, 1963), p. 393. "Currency appreciation is a deflationary device in an economy with idle resources."

[32] Egon Sohmen, Flexible Exchange Rates (Chicago: University of Chicago Press, 1961), p. 26. "A capital inflow is consequently directly deflationary under fluctuating exchange rates . . . ".

[33] Ibid., p. 99.

or contractionary,[34] and Machlup,[35] in an excellent review of the literature on the effects of devaluation, warns that we should avoid drawing definite conclusions from algebraic models which cannot take account of all the variables.

While this has by no means been an exhaustive review of the literature on the effects of capital transfers, it is, perhaps, sufficient to illustrate the current opinion. We shall now proceed to set up a model in order that this problem can be more closely examined.

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[34]John Burr Williams, International Trade Under Flexible Exchange Rates (Amsterdam: North-Holland Publishing Company, 1954).

[35]Fritz Machlup, "The Terms-of-Trade Effect of Devaluation Upon Real Income and the Balance of Trade," Kyklos, IX (1961), 417-452.

CHAPTER II

A CAPITAL TRANSFER MODEL

1. Introduction

This chapter consists of two sections. In the first, all the assumptions underlying our model will be made explicit, and some of the theoretical problems involved in formulating a mathematical model will be discussed. In the second, the model itself will be developed and a brief discussion of stability conditions will be made.

2. The Assumptions of the Model

The model is first of all Keynesian, and is in real rather than money terms. Thus national income, the marginal propensity to import, the marginal propensity to spend, and all other variables and parameters are in real terms. This assumption is necessary for two reasons. First of all, since we are interested in changes in real income and employment, we must work in real terms if we are to avoid fluctuations due to changing prices which would distort our results. Secondly, as we shall see later, working in real terms allows us to draw a parallel between real income and employment.

The changes in national income shown by our model

are assumed to be real. Since changes in other variables of the Keynesian system could invalidate this assumption, we must make further assumptions in order that these variables have a neutral effect. We must, in other words, assume conditions which will prevent domestic prices from changing. Prices here include the interest rate (the price of money), the wage rate (the price of labour), and the domestic commodity price level.[1] The interest rate will be constant if the economy is in such a depressed state that we are, both before and after a change in income, in the Keynesian "liquidity trap". Here the demand for money is perfectly interest-elastic, and thus the liquidity preference curve is horizontal and the interest rate cannot change. The interest rate would also be constant if it were stabilized by the monetary authorities. This assumes that they are prepared to buy or sell any amount of currency at some given price, i.e., the supply of money is completely elastic.

If either of these conditions is fulfilled, the interest rate will have no effect on savings or investment, and thus these two variables must depend on the levels of real income and price only. This condition would also be

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[1] This constancy of the price level applies only to domestic prices and does not include the prices of import goods. It is possible (at least theoretically) for domestic prices to remain constant and the general price level to change, for the prices of imports must be included in the general price level. The rather unrealistic assumption that domestic prices could remain constant while import prices change is discussed below.

satisfied if we assumed that both savings and investment were perfectly interest-inelastic. Most economists, however, would consider such a situation highly unlikely, and we will therefore ignore this possibility.[2]

The constancy of the price level depends ultimately on the constancy of marginal productivity. If the demand for some product or products increases, prices can remain constant only if more can be produced at the same average cost.[3] This means that the supply curves of all the factors of production, including labour, must be infinitely elastic. An infinitely elastic supply curve implies unused capacity; in the case of labour--unemployment. Thus, constant marginal productivity, implying unused capacity, is enough, ceteris paribus, to insure that commodity prices (of domestically produced goods) will remain constant. At the same time, unused capacity in the labour force (unemployment) insures that aggregate employment can be increased without bidding up the real wage rate.

The constancy of all prices allows us, as mentioned earlier, to form a direct link between changes in employment and changes in national income, for if the wage rate

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[2] For example, Jaroslav Vanek, International Trade: Theory and Economic Policy (Homewood, Illinois: Richard D. Irwin, Inc., 1962), p. 104, considers interest-inelastic investment and savings schedules to be a "quite unrealistic" assumption.

[3] Or, of course, at the same marginal cost, for if the average cost is constant, the average cost and the marginal cost must be equal.

is constant, and if marginal productivity is constant, then output cannot be increased without increasing employment. Employment here, of course, must be in aggregate terms, such as total man-hours or total man-years. This argument also assumes that all output has some labour content and that we have fixed factor proportions in all industries.

While the relationship between employment and national income is direct, we cannot assume it to be fixed, for this would imply that all output is produced with a constant proportion of labour, an assumption which is not likely to be realized. Thus while we can say that when real national income increases, employment increases, we cannot say by how much, nor can we say that the increases will always be proportionally the same. An increase in national income of ten percent might increase employment five percent at one time and fifteen percent at another, depending on in which sector of the economy the increase in output takes place. For the purposes of our analysis, the labour force and the population will be assumed constant, so that these factors cannot distort our results.

Throughout this analysis we have ignored such things as innovations, institutional factors such as labour unions, effects which a redistribution of income might have, and all other influences which could complicate this simple framework. We have also assumed away many of the things which we criticised other writers for not

mentioning. For example, Machlup's "resource-reallocation" effect has been nullified because of our assumption of constant domestic prices. Theoretically, it is highly unlikely that domestic prices will remain constant if import prices change, unless, of course, there is no substitution between imports and domestic goods. If this is not the case then the prices of import-competing goods will tend to follow the prices of imports, otherwise domestic producers would be forced out of business. Similarly, if the foreign prices of our exports rise (fall) there will be upward (downward) pressure on the prices of these goods in the domestic market.

We have similarly assumed away any monetary effects, by assuming that the interest rate will remain constant. On theoretical grounds, it is clearly unrealistic to think that capital inflows will not exert pressure on interest rates, and equally as unrealistic to think that the monetary authorities will succeed in neutralizing, or for that matter attempt to neutralize, this pressure. The action that will be taken, of course, depends, first of all, on the economic conditions of the country at the time. This we have dealt with by assumption. Secondly, the action taken will depend on what the authorities feel the effects of capital inflows are, the degree to which they think monetary action can control capital inflows, and the amount of confidence they have in monetary tools for controlling the interest rate and the money supply. One

answer to the first of these was given by Mr. Coyne, as is shown in Chapter I. The second will depend on how important interest rate differentials are deemed to be as causes of capital inflows, which in turn, will depend on the way different types of capital inflow, i.e., direct or portfolio, are thought to respond to the interest rate, and on the relative magnitude of each. The third of these does not appear to have any definite answer either, for Mr. Coyne indicated that he did not feel that the interest rate could always be changed by monetary policy.[4] Thus, while the assumption of a constant interest rate is not free from attack, it is necessary if our model is to be simple enough to handle, and while it may be unrealistic, it is probably less unrealistic than any other assumption that could be made which, at the same time, would facilitate analysis.

Our model also does not take account of those indirect effects which we mentioned earlier. We cannot take account in our formal mathematical model of the changes in

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[4]See, for example, James E. Coyne, "Inflation and Unemployment". Remarks delivered at the 48th Annual Meeting of the Ontario Chamber of Commerce in Hamilton, May 12, 1960 (Mimeographed by the Bank of Canada). Mr. Coyne says that there are some who suggest "that there should be a large degree of monetary expansion in Canada with a view to bringing about a reduction in interest rates in Canada and making more money available for lending at lower rates of interest, so that Canadian borrowers would not feel under any inducement to borrow in New York. This proposal has many drawbacks, including the obvious dangers of inflation, associated with it. But in addition to all other handicaps it suffers in my opinion from the disability that it simply would not work." p. 12.

the savings and investment plans which a capital inflow might cause. No account can be taken of changes in expectations, nor of the indirect employment effects which capital inflows might have if used for direct investment. Since we cannot take account of these indirect employment effects we must implicitly be assuming that they do not exist. That is, we are assuming either that capital inflows used for investment have no indirect employment effects, or that none of the capital inflow is used for direct investment purposes. Since we want to examine the effects of different types of capital flows, we must reject the latter and accept the former, knowing, of course, that it will very probably be violated.

Two kinds of problems arise if all or part of the capital inflow is used to buy imports, and if these imports are either machinery or raw materials, for such imports are intermediate goods and they must have an effect on domestic output. The first of these has to do with the exclusion of the import content from domestic output. This problem, however, while bothersome empirically, does not present any difficulty conceptually, and therefore, need not concern us further. The second involves the imports of factors of production, and is somewhat more troublesome. As Machlup says:

> The measurement of changes in real domestic output is complicated by the fact that imported raw materials and imported machines are used in domestic production and must conceptually and statistically be excluded from domestic output. A builder of a simple analytical

> model will save himself much trouble by assuming that imports consist only of finished consumers goods.[5]

Unfortunately, we cannot take Machlup's advice, for we know that many of our imports are machines, which add to domestic output. Both conceptually and statistically, it would be difficult to exclude this contribution from domestic output. For our purposes we will assume, then, that this factor does not exist. This will in no way affect the validity of our results, for our main concern is with the level of employment, and employment is certainly more closely associated with the total output of Canada than it is with total output of Canadian factors, that is, more closely associated with gross national product than with gross domestic product.

From the above discussion we see that our model does not take account of many indirect factors which theoretically could have some effect on changes in real national income. Our model also assumes away, either explicitly or implicitly, a number of direct effects which could influence the level of national income. This does not mean, of course, that we are ignoring these things. It simply means that since our model is unable to take account of them, we must consider them in conjunction with, rather than as a part of, our model. The conclusions of our analysis will thus have to be tempered with all these other factors.

We will now turn from a discussion of the assumptions

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[5] Machlup, loc. cit., p. 440.

of our model, to a discussion of some of the definitional and conceptual problems involved in formulating the model. The first two of these relate to our most fundamental variables; national income and capital inflows.

While we have already discussed the problem of real versus money national income, the concept of national income itself deserves further consideration. In discussing the vagueness of the national income concept Machlup distinguishes three concepts; real domestic output, real intake, and real income, from the "large family of magnitudes related to social accounting".[6] These he defines as follows:

> (1) . . . the total domestic production of finished goods and services valued at constant prices ("real domestic output"), (2) . . . the total domestic use for consumption and investment of goods and services valued at constant prices ("real intake"), and (3) . . . the total amount of income received or earned, with goods and services valued at constant prices, and with income transfers, capital earnings, and all changes in foreign assets and liabilities accounted for insofar as they arise from current transactions ("real income") . . .[7]

He then goes on to say that real intake will be greater than real output if an import balance exists, and that an export balance implies that real intake is less than real output. The difference between real output and real income, however, is found only in some specific instances, such as;

> when reparation payments and gifts are counted as income of the receiving country; when certain earnings

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[6] Ibid., p. 439. [7] Ibid.

> from foreign investment are included in the income of the receiving country but not deducted from the real domestic output of the paying country; when the commodity terms of trade have changed and the change is reflected partly or fully in a loss of foreign assets or an increase of foreign debts.[8]

Haberler makes these distinctions clearer by the use of identities.[9] He defines the terms national income (Y), consumption (C), domestic investment (I_d), exports (X), imports (M), income from foreign investment (D), and reparations and gifts (R). He is then able to write:

total expenditure, $V = C + I_d$

volume of production, $P = C + I_d + X - M$

national income, $Y = C + I_d + X - M + D + R$.

Haberler's national income, total expenditure, and volume of production correspond respectively to Machlup's real income, real domestic output, and real intake. These identities clearly indicate the relationships among the three concepts. We see, as Machlup pointed out, that total expenditure and the volume of production differ by the amount of the commodity trade balance, and that national income differs from the volume of production by interest and dividend payments, reparations, and gifts.[10] One other

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[8] Ibid., p. 441.

[9] Gottfried Haberler, A Survey of International Trade Theory (Princeton, New Jersey: Princeton University Press, 1961).

[10] Machlup's extra term in his definition of national income is due to the fact that he defines his terms differently, because he wishes to deal with the problems of changes in foreign assets and liabilities, while Haberler's concern is with such things as reparation payments, gifts, and foreign aid. Since this difference is of no significance to our analysis, we will not deal further with it here.

point worth noting is that in the above analysis;

> consumption (C) and domestic investment (I_d) are defined so as to include imported consumption and investment goods. In the theoretical literature, on the other hand, C and I_d are frequently defined as home produced consumption and investment goods. It is difficult, however, to implement the latter distinction statistically.[11]

Our model suffers from this disadvantage, and if statistical estimation were to be undertaken, a redefinition of the variables would be necessary.[12]

The "national income" of the model developed in this chapter is not national income as defined above, but is Machlup's real domestic output, for we take no account of interest and dividend payments, reparations, gifts or foreign aid. We now have two courses open. We can continue to use the term national income and understand that it is actually real domestic output, or we can change real domestic output into national income by in some way including interest and dividend payments, reparations, gifts and foreign aid. If we choose the former, however, we are forced to include dividends, interest, and the rest, somewhere else in our model, for if we do not, we are, in fact, implicitly assuming that they do not exist. Since they are not elsewhere included in the model, we must devise

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[11] Ibid., p. 32.

[12] For a model similar to the one used here, but where consumption and investment are defined to include imports, see Svend Laursen and Lloyd A. Metzler, "Flexible Exchange Rates and the Theory of Employment," The Review of Economics and Statistics, XXXII (November, 1950), 281-299.

some other way of dealing with them. We could, of course, simply assume that they are non-existant. This, however, would be in contradiction to our other assumptions, for in a model which is built around capital flows, income from foreign investments must surely exist. We can perhaps skirt the problem by redefining national income and capital flows. Since we are primarily interested in the effects of changes in income on the level of employment, and since income earned abroad does not affect employment, our analysis will not suffer from the exclusion of interest and dividends from national income. Now if we assume that these interest and dividend payments actually do take place, they become a type of capital transfer, and thus we can include them in our definition of net capital flows. The other parts of this group, that is, foreign aid, reparations, and gifts, we can certainly assume to be non-existant. Thus the problem is "solved" by defining national income to exclude, and capital flows to include, payments of interest and dividends from foreigners. It must be pointed out, that by combining interest and dividends with capital flows, we are combining a current account item and a capital account item, a not entirely desirable practise. Our main concern here, however, is whether interest and dividend payments are exogenous or endogenous to our system. As we shall see in section 3 of this chapter, we include in our model a shift variable which includes all autonomous changes, including changes

in the level of capital flow. All the other "variables" are functions of income and price. Interest and dividends are not usually considered to be functionally related to income or price, so they are best combined with capital flows and included in the autonomous shift variable.

This brings us logically to a discussion of capital flows; the most important issue here being whether capital flows are autonomous or induced. By autonomous capital inflows we mean inflows which occur either because foreigners wish to invest in Canada (perhaps because of better investment opportunities) or because Canadians prefer to borrow abroad (perhaps because of lower foreign interest rates). By induced capital flows, we mean flows which are a direct response to exports and imports. For example, if a country's imports exceed its exports, it must pay for the difference by borrowing foreign currency. This is an induced capital inflow.

This distinction between autonomous and induced capital flows is important, because induced capital flows do not, themselves, represent a force which would bring about changes in national income. They are simply responses to already existing forces. As Machlup has said in discussing such flows:

> Thus, this foreign lending (or gold purchase) must not be considered as a separate force, but rather as a response to, or perhaps an integral part of, the force exerted by the increase in exports.[13]

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[13] Fritz Machlup, International Trade and the National Income Multiplier (Philadelphia: The Blakiston Company, 1943), p. 132.

Thus, if we are to show how capital movements change national income, we must assume that the capital movements of which we speak are autonomous rather than induced. While this is a rather heroic assumption, there does seem to be some justification for it, at least for the Canadian situation. On this problem Machlup has said:

> The question of whether capital movements lead the trade balances, or trade balances direct the capital movements, has given rise to much discussion and it is somewhat bewildering that experts should arrive at opposite answers.[14]

He then goes on to explain that the classical view was that the trade balance adjusted to capital flows, and lists such persons as Hume, Thornton, Ricardo, J. S. Mill and Viner as among those who held this opinion. The opposite view, that the capital flows adjusted to trade balance was made popular by Keynes, "and, hence, it is now very widely held everywhere ".[15] Viner has expressed the opinion that "there is no apparent a priori reason why the dependence should not be as much in one direction as the other."[16] He goes on to say, however, that "Examination of such data as are readily available strongly confirms

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[14] Ibid., p. 136.

[15] Ibid., p. 136. n. On this subject Keynes said "Historically, the volume of foreign investment has tended, I think, to adjust itself--at least to a certain extent--to the balance of trade, rather than the other way round, the former being the sensitive and the latter the insensitive factor." John Maynard Keynes, "The German Transfer Problem," Economic Journal, XXXIX (March, 1929), p. 6.

[16] Jacob Viner, Studies in the Theory of International Trade (New York: Harper and Bros., 1937), p. 364.

however, the orthodox doctrine . . . that major long-term capital movements have . . . mainly been 'disturbing' rather than 'equilibrating' in nature."[17] "Disturbing" here means autonomous while "equilibrating" means induced.

Machlup concludes his discussion by saying that capital flows are likely more often a cause than an effect of a trade balance, and suggests that this impression is strengthened by the empirical work which has been done. He refers to Professor Viner's research and says:

> Professor Viner, after a painstaking examination of the available data, takes Canada from 1900-1913 as an example of a case where foreign borrowing can be considered as the cause of the import surpluses. The evidence is fully convincing . . .[18]

While we cannot conclude from this that the capital inflows of the 1950's were the cause of the import surplus, a casual comparison seems to indicate that there is a definite similarity between the two periods, and in particular between the types of capital inflows of these periods.[19] We must also remember that Machlup's analysis and Viner's investigation were carried on under fixed exchange rate conditions, and the conclusions would probably be much stronger if flexible exchange rates had existed. The criticism of this theory, particularly as expressed by Keynes, was that capital flows tend to be much more flex-

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[17]Ibid., p. 365.

[18]Machlup, op. cit., p. 143, n.2.

[19]This is particularly so if we compare 1900-1913 with 1950-1957.

ible than trade balances and it is therefore more likely that capital flows will adjust to the balance of imports and exports than the other way around. This argument loses much of its force when flexible rates are considered, for here the value of exports and imports is much more easily adjusted via the exchange rate. Thus it seems logical to assume that long term capital flows have been the cause, rather than the effect, of income changes.

The term "capital flows", as we have used it up to now, has been very vague, and we must define exactly what the term is to mean. It includes only net long-term capital flows and, as was mentioned above, is defined to include payments of interest and dividends to and by foreigners. Excluded are any type of short term capital movements, and initially these are assumed to be absent. Short term capital movements are considered as outside forces which could affect the results of our theoretical model. They cannot, however, be included in the model itself. The problem of differentiating between long and short term capital movements, while difficult empirically, need not bother us at the theoretical level. We need only assume that such a differentiation can be made.

There are a number of other controversial issues which we must deal with before proceeding to set up our model. One of these involves the applicability of Keynesian tools to the problems of today, and especially to those in the field of international trade. Sohmen, for

example, suggests that ". . . there is every indication that the Keynesian variety of unemployment is hardly a live problem any more."[20] He later goes on to say, "Keynesian tools reveal their intrinsic weakness perhaps more clearly when they are applied to problems of international trade than for any other use."[21] Sohmen's suggestion that "Keynesian type of unemployment" is not a real issue does not seem to apply to the Canadian situation, for during the late 1950's our level of unemployment occasionally went as high as ten percent. Only if this were of a type which could not be decreased by increasing effective demand, (i.e., structural) could Sohmen's contention be true.

Sohmen's argument that Keynesian tools are inappropriate for international trade problems is a much more serious charge and unfortunately much less easily refuted. His argument is, that when exchange rates are flexible, many of the concepts usually employed become very ambiguous, particularly the marginal propensity to import and national income. When the exchange rate changes it is not at all clear what real imports are, nor for that matter, what real income is. He goes on to say that:

> The combination of the various possibilities of defining imports and real income yields a number of "marginal propensities to import," none of which has self-evident claim to being the most natural one.[22]

While this is certainly true, the shortcomings of some of our Keynesian concepts may not be as serious as Sohmen claims. In the first place, differences in propensities

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[20]Sohmen, op. cit., p. 43.

[21]Ibid., p. 44. [22]Ibid.

due to differences in definitions are likely to be relatively small and unimportant, especially in a model such as ours where exact values are not essential. When these differences are put up alongside the problems involved in getting any sort of statistical measurement of the propensities, they will seem small indeed. Secondly, these sorts of problems can be minimized by exactly defining what we mean by these terms. Thus, while we can be attacked for using the "wrong" national income concept or the "wrong" marginal propensity to import, we cannot be attacked for having left these terms ambiguously defined. One further defense of Keynesian tools can be made. While they may be imperfect, no other comparably efficient tools exist, and thus if an analysis is to be made, Keynesian methods seem to be our only choice.

Sohmen further goes on to say that:

> . . . the assumption of constancy of <u>any</u> one of these marginal propensities even for given values of real national income (however defined) will generally conflict with basic norms of economic behavior whenever flexibility of exchange rates is admitted. . . . Marginal import propensities will inevitably be affected by variations of exchange rates.[23]

This is a valid criticism and is one of the reasons why we plan to examine the effects on the change in national income of changes in the elasticities and propensities. This can be done by calculating a set of answers given some assumed values for the parameters, and then changing these values, one at a time, and recalculating the results.

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[23]<u>Ibid</u>.

We must not, of course, confuse this procedure with that of deriving expressions for changes in national income with respect to changes in the elasticities and propensities, i.e., differentiating with respect to these parameters. What we, in fact, are doing, is assuming that these paramaters are constant, but that we do not know their exact values. This is probably an oversimplification, for it is much more likely that the elasticities and propensities are functionally related to national income and price.

Another difficulty, common to all aggregate models, is the index number problem. We talk about wage rates and price levels as if they could be uniquely and unambiguously determined, while, in fact, they are some sort of average which can be calculated in any number of ways. This is particularly important in the case of the commodity prices, for we have assumed that a change in the exchange rate, by changing prices of foreign goods, will change the price level. Problems, such as what weights to use--those before the change, those after, or some average of these--arise when any attempt is made to estimate the changes in the price level. These problems, however, similar to those of long-term versus short-term capital flows, while very bothersome empirically, need not concern us at the theoretical level. It is sufficient to assume that such indices can be found.

Professor Harberger, in a review of Stuvel's book,[24] lists two common pitfalls in mathematical model-building.[25] He calls these the "fallacy of completeness" and the "problem of translation". Discussing the "fallacy of completeness", he says that when economists formulate models which are mathematically complete, they are liable to assume that this insures that their models are complete in every respect, and therefore, that results that might be computed by using the model will necessarily be correct. This, of course, ignores all the things that the model cannot possibly include if it is to be workable; much the same kind of problem that we discussed above.

The "problem of translation" is the problem of translating economic language into mathematical language. Some concepts, such as elasticity of demand, marginal utility, and the multiplier, are easily translated, while others, such as the concept of stability, are not. The stability of a model will depend on the underlying assumptions, and therefore, showing that a model is stable only proves that it is stable for the specific set of assumptions we have used. It may be quite unstable under a different set of assumptions. This fact is often overlooked, and consequently conclusions are reached which are unjustifiable in any general sense. More will be said

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[24]G. Stuvel, The Exchange Stability Problem (New York: Augustus M. Kelley, 1951).

[25]Arnold C. Harberger, "Pitfalls in Mathematical Model Building," American Economic Review, XLII (December, 1952), 856-865.

about stability at the end of the next section. Now, keeping all the above assumptions and qualifications in mind, we will proceed to formulate our model.

3. The Model[26]

We assume that there are only two countries, country 1 and country 2, and that both national income and relative prices are variable. The exchange rate is also assumed to be flexible so that the foreign-exchange market is always permitted to adjust to its equilibrium level. We assume, as mentioned above, that both economies are operating at less than full employment where the interest rate is fixed and where marginal productivity is constant. We will assume that prices in both economies are equal to unity. If, in reality, they are not, we can redefine the quantity measurements to make them so. Our variables can now be expressed as depending on two factors; real national income and relative prices. Because we have set prices equal to unity, differences in price levels will occur only with changes in the exchange rate. For example, demand for imports from country 1 by country 2, will be a function of the income of country 2 and the exchange rate. We will define by r the number of units of country 1's currency paid for one unit of country 2's currency. The demand by country 2 for the output of country 1 (country 2's imports) will then be a function of Y_2

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[26]The formulation of this model follows Vanek quite closely. See Vanek, op. cit., especially chap. 8.

(the income of country 2), and 1/r (the price of country 2's imports). Similarly the demand by country 1 for the output of country 2 will be a function of Y_1 and r. Now, denoting exports of country 1 by m_{12} and exports of country 2 by m_{21}, we can write

$$m_{12} = m_{12}\left(Y_2, \frac{1}{r}\right) \quad \text{- - - - - - - - - - - - - - - - - - - (1)}$$

and

$$m_{21} = m_{21}\left(Y_1, r\right) \quad \text{- - - - - - - - - - - - - - - - - - (2)}$$

The first subscript in the above refers to the supplying country and the second to the demanding country. This notation will be used throughout the analysis. By this same notation we can now define two other variables, m_{11} and m_{22}, showing the demand in each country for domestically produced goods. These, in turn, will be functions of national income and the exchange rate, because when the exchange rate changes, prices of domestic output remaining constant, foreign goods will become relatively more or less expensive and substitution will take place. We may thus write

$$m_{11} = m_{11}\left(Y_1, r\right) \quad \text{- - - - - - - - - - - - - - - - - - - (3)}$$

and

$$m_{22} = m_{22}\left(Y_2, 1/r\right). \quad \text{- - - - - - - - - - - - - - - - - (4)}$$

The terms m_{11} and m_{22} represent total absorption of domestically-produced goods, and include consumption, investment, and government spending.[27]

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[27] This assumes, of course, that each of these is a function of national income and the price level. If any of these are not functions of income and price then they will not be included.

Now, because real national income of a country consists of two parts, real absorption and real exports, we can write

$$Y_1 = m_{11}(Y_1, r) + m_{12}(Y_2, 1/r) + pa \text{ - - - - - - -} \quad (5)$$

and

$$Y_2 = m_{21}(Y_1, r) + m_{22}(Y_2, 1/r) + qa \text{ - - - - - - -} \quad (6)$$

The "a"s are shift variables which represent autonomous changes in any of the components of Y. Thus, they could represent changes in consumption, investment, government spending, or exports. p and q are constants which will be explained later.

Along with equations (5) and (6) we now need an equation expressing the equality of exports and imports in terms of one currency or the other. We can express this as

$$0 = m_{21}(Y_1, r)\cdot r - m_{12}(Y_2, 1/r) - va \text{ - - - - - - -} \quad (7)$$

where, as before, v is a constant, and "a" is a shift variable representing changes in m_{12}, i.e., exports of country 1. In the following analysis this "a" will also be used to represent changes in capital flows, for as was pointed out in Chapter I, "an autonomous inflow of funds, for example, increases the supply of foreign exchange exactly as if exports of goods and services had increased by the same amount."[28] It could be argued that equation (7) should include a term specifically to represent capital flows, i.e.,

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[28] Sohmen, op. cit., p. 19.

$$0 = m_{21}(Y_1, r)\cdot r - m_{12}(Y_2, 1/r) - C - va$$

where "C" represents capital inflows. While this would certainly be acceptable, it would complicate our analysis unnessarily, for changes in capital inflows can be included in the "a" term and in the initial equilibrium, net capital flows are assumed to be zero.

These three equations are sufficient to determine our three variables, Y_1, Y_2, and r. Once r has been found, "physical units of measurement and prices in one country or the other may be redefined in such a way as to yield r = 1."[29] Thus we will assume for the rest of the analysis that the exchange rate is initially equal to 1.

We can now examine the effect of a change in "a" by differentiating equations (5), (6) and (7). Taking the total derivative of equation (5), and writing $1/r = r^*$, we have

$$dY_1 = \frac{\partial m_{11}}{\partial Y_1} dY_1 + \frac{\partial m_{12}}{\partial Y_2} dY_2 + \frac{\partial m_{11}}{\partial r} dr + \frac{\partial m_{12}}{\partial (r^*)} dr^* + pda$$

dividing by da

$$\frac{dY_1}{da} = \frac{\partial m_{11}}{\partial Y_1}\frac{dY_1}{da} + \frac{\partial m_{12}}{\partial Y_2}\frac{dY_2}{da} + \frac{\partial m_{11}}{\partial r}\frac{dr}{da} + \frac{\partial m_{12}}{\partial (r^*)}\frac{d(r^*)}{da} + p$$

or

$$\frac{dY_1}{da} - \frac{\partial m_{11}}{\partial Y_1}\frac{dY_1}{da} - \frac{\partial m_{12}}{\partial Y_2}\frac{dY_2}{da} - \frac{\partial m_{11}}{\partial r}\frac{dr}{da} - \frac{\partial m_{12}}{\partial (r^*)}\frac{d(r^*)}{da} = p.$$

Now $\frac{\partial m_{11}}{\partial Y_1}$ is the marginal propensity to spend in country 1, and $\frac{\partial m_{12}}{\partial Y_2}$ is the marginal propensity to import of country 2. We will write these as m'_{11} and m'_{12} respectively.

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[29] Vanek, op. cit., p. 138.

Now, changing $\frac{dr^*}{da}$ to $\frac{dr^*}{dr}\frac{dr}{da}$, and remembering that $r^* = 1/r$, we can write $\frac{dr^*}{da} = (-\frac{1}{r^2})\frac{dr}{da}$. Our equation now becomes

$$(1 - m'_{11})\frac{dY_1}{da} - m'_{12}\frac{dY_2}{da} - \frac{dr}{da}\left[\frac{\partial m_{11}}{\partial r} - \frac{1}{r^2}\frac{\partial m_{12}}{\partial (r^*)}\right] = p.$$

Now, multiplying both numerator and denominator of the first term of the bracketed expression by m_{11} and the second term by m_{12} and remembering that we have defined $r = 1$, we can write

$$(1 - m'_{11})\frac{dY_1}{da} - m'_{12}\frac{dY_2}{da} - \frac{dr}{da}\left[\frac{\partial m_{11}}{\partial r}\frac{r}{m_{11}}m_{11} - \frac{\partial m_{12}}{\partial (r^*)}\frac{(r^*)}{m_{12}}m_{12}\right] = p.$$

Here, $\frac{\partial m_{12}}{\partial (r^*)}\frac{(r^*)}{m_{12}}$ is country 2's elasticity of demand for imports, which we will write as d_{12}, and $\frac{\partial m_{11}}{\partial r}\frac{r}{m_{11}}$ is a cross elasticity of demand for domestic output, and shows the change in spending on home produced goods associated with a change in the price of foreign goods, as reflected in the exchange rate. We will call this term d^+_{11}. We can now write

$$(1 - m'_{11})\frac{dY_1}{da} - m'_{12}\frac{dY_2}{da} - [d^+_{11}m_{11} - d_{12}m_{12}]\frac{dr}{da} = p$$

or

$$(1 - m'_{11})\frac{dY_1}{da} - m'_{12}\frac{dY_2}{da} - A_1\frac{dr}{da} = p \quad \text{- - - - - - - - - - (8)}$$

where

$$A_1 = [d^+_{11}\ m_{11} - d_{12}\ m_{12}].$$

Differentiating equation (6) we have

$$dY_2 = \frac{\partial m_{21}}{\partial Y_1}dY_1 + \frac{\partial m_{22}}{\partial Y_2}dY_2 + \frac{\partial m_{21}}{\partial r}dr + \frac{\partial m_{22}}{\partial (r^*)}r^* + qda$$

dividing by da

$$\frac{dY_2}{da} = \frac{\partial m_{21}}{\partial Y_1}\frac{dY_1}{da} + \frac{\partial m_{22}}{\partial Y_2}\frac{dY_2}{da} + \frac{\partial m_{21}}{\partial r}\frac{dr}{da} + \frac{\partial m_{22}}{\partial (r^*)}\frac{d(r^*)}{da} + q.$$

Rewriting the marginal propensities to spend and to import as before, we have

$$\frac{dY_2}{da} - m'_{21}\frac{dY_1}{da} - m'_{22}\frac{dY_2}{da} - \frac{\partial m_{21}}{\partial r}\frac{dr}{da} - \frac{\partial m_{22}}{\partial (r^*)}\frac{d(r^*)}{da} = q.$$

Again, writing $\frac{d(r^*)}{da} = (-1/r^2)\frac{dr}{da}$ and collecting terms

$$(1 - m'_{22})\frac{dY_2}{da} - m'_{21}\frac{dY_1}{da} - r^*\frac{dr}{da}\left[\frac{\partial m_{21}}{\partial r}\frac{r}{} - r^*\frac{\partial m_{22}}{\partial r^*}\right] = q.$$

Multiplying numerator and denominator of the first term of the bracketed expression by m_{21} and the second by m_{22}, and remembering that $r = 1$, we can write

$$(1 - m'_{22})\frac{dY_2}{da} - m'_{21}\frac{dY_1}{da} - \left[\frac{\partial m_{21}}{\partial r}\frac{r}{m_{21}}m_{21} - \frac{\partial m_{22}}{\partial r^*}\frac{r^*}{m_{22}}m_{22}\right]\frac{dr}{da} = q$$

or, as before

$$(1 - m'_{22})\frac{dY_2}{da} - m'_{21}\frac{dY_1}{da} - [d_{21}\,m_{21} - d^+_{22}\,m_{22}]\frac{dr}{da} = q$$

$$(1 - m'_{22})\frac{dY_2}{da} - m'_{21}\frac{dY_1}{da} - A_2\frac{dr}{da} = q \qquad (9)$$

where $A_2 = [d_{21}\,m_{21} - d^+_{22}\,m_{22}]$ and where d_{21} is country 1's elasticity of demand for imports and d^+_{22} is country 2's cross elasticity of demand for domestically produced goods with respect to the price of imports. Similarly, for equation (7)

$$0 = r\cdot\frac{\partial m_{21}}{\partial Y_1}dY_1 - \frac{\partial m_{12}}{\partial Y_2}dY_2 + r\cdot\frac{\partial m_{21}}{\partial r}dr + m_{21}\,dr - \frac{\partial m_{12}}{\partial r^*}dr^* - v\,da.$$

Dividing by da

$$- r \frac{\partial m_{21}}{\partial Y_1} \frac{dY_1}{da} + \frac{\partial m_{12}}{\partial Y_2} \frac{dY_2}{da} - r \frac{\partial m_{21}}{\partial r} \frac{dr}{da} - m_{21} \frac{dr}{da} + \frac{\partial m_{12}}{\partial r^*} \frac{dr^*}{da} = -v.$$

Defining our propensities as before, collecting terms, and writing $\frac{dr^*}{da} = (-\frac{1}{r^2}) \frac{dr}{da}$ we have

$$- r\, m'_{21} \frac{dY_1}{da} + m'_{12} \frac{dY_2}{da} - \frac{dr}{da} [r \frac{\partial m_{21}}{\partial r} + m_{21} + \frac{1}{r^2} \frac{\partial m_{12}}{\partial r^*}] = -v$$

or

$$- r\, m'_{21} \frac{dY_1}{da} + m'_{12} \frac{dY_2}{da} - r^* \frac{dr}{da} [\frac{\partial m_{21}}{\partial r} r^2 + m_{21} r + r^* \frac{\partial m_{12}}{\partial r^*}] = -v.$$

Now, multiplying numerator and denominator of the first term of the bracketed expression by m_{21} and the third by m_{12} and remembering that r = 1, we can write

$$- m'_{21} \frac{dY_1}{da} + m'_{12} \frac{dY_2}{da} - [\frac{\partial m_{21}}{\partial r} \frac{r}{m_{21}} m_{21} + m_{21} + \frac{\partial m_{12}}{\partial r^*} \frac{r^*}{m_{12}} m_{12}] \frac{dr}{da} = -v$$

or

$$- m'_{21} \frac{dY_1}{da} + m'_{12} \frac{dY_2}{da} - [m_{21} + d_{21} m_{21} + d_{12} m_{12}] \frac{dr}{da} = -v$$

$$- m'_{21} \frac{dY_1}{da} + m'_{12} \frac{dY_2}{da} - A_3 \frac{dr}{da} = -v \quad - - - - - - - - - (10)$$

where $A_3 = [m_{21} + d_{21} m_{21} + d_{12} m_{12}]$, and d_{21} and d_{12} are defined as before. We have now the three equations

$$(1 - m'_{11}) \frac{dY_1}{da} - m'_{12} \frac{dY_2}{da} - A_1 \frac{dr}{da} = p$$

$$- m'_{21} \frac{dY_1}{da} + (1 - m'_{22}) \frac{dY_2}{da} - A_2 \frac{dr}{da} = q$$

$$- m'_{21} \frac{dY_1}{da} + m'_{12} \frac{dY_2}{da} - A_3 \frac{dr}{da} = -v$$

or, written in matrix notation,

$$\begin{bmatrix} (1 - m'_{11}) & - m'_{12} & - A_1 \\ - m'_{21} & (1 - m'_{22}) & - A_2 \\ - m'_{21} & + m'_{12} & - A_3 \end{bmatrix} \begin{bmatrix} \frac{dY_1}{da} \\ \frac{dY_2}{da} \\ \frac{dr}{da} \end{bmatrix} = \begin{bmatrix} +p \\ +q \\ -v \end{bmatrix} \quad --(11)$$

This, now is the most general form of our model. Any change in the spending of either country can be analysed by assigning appropriate values to p, q and v, and solving for dY_1, dY_2 and dr. Thus, if we assumed that the demand in country 2 for imports from country 1 autonomously increased, and that this had no effect on the demand for the output of country 2, then the right-hand vector of (11) would become

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} .$$

If, on the other hand, we assume that this increase in demand for imports in country 2 decreased demand for home produced goods by the same amount, then the right-hand vector would be

$$\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} .$$

There are, of course, an unlimited number of other possibilities.

Solving equation (11) for dY_1 we get

$$\frac{dY_1}{da} = \frac{1}{E}\left[- pA_3(1 - m'_{22}) - A_2 v\, m'_{12} - A_1 q\, m'_{12} - A_3 q\, m'_{12} + A_2 p\, m'_{12} - A_1 v\,(1 - m'_{22})\right]$$

$$dY_1 = \frac{da}{E}\left\{ p\left[A_2 m'_{12} - A_3(1 - m'_{22})\right] - q\, m'_{12}\left[A_1 + A_3\right] - v\left[A_2 m'_{12} + A_1(1 - m'_{22})\right]\right\}.$$

Where E is the value of the coefficient determinant, and can be written

$$E = \left[-A_3(1 - m'_{22})(1 - m'_{11}) - A_2 m'_{12} m'_{21} + A_1 m'_{21} m'_{12} + A_3 m'_{21} m'_{12} + A_2 m'_{12}(1 - m'_{11}) - A_1 m'_{21}(1 - m'_{22})\right]$$

$$E = A_1\left[m'_{12} m'_{21} - m'_{21}(1 - m'_{22})\right] - A_2\left[m'_{12} m'_{21} - m'_{12}(1 - m'_{11})\right] - A_3\left[(1 - m'_{22})(1 - m'_{11}) - m'_{21} m'_{12}\right].^{30}$$

Now, solving for dY_2

$$dY_2 = \frac{da}{E}\left[-A_3 q\,(1 - m'_{11}) + A_2 p\, m'_{21} - A_1 v\, m'_{21} - A_3 p\, m'_{21} - A_2 v\,(1 - m'_{11}) - A_1 q\, m'_{21}\right]$$

$$dY_2 = \frac{da}{E}\left\{ p\, m'_{21}\left[A_2 - A_3\right] - q\left[A_1 m'_{21} + A_3(1 - m'_{11})\right] - v\left[A_1 m'_{21} - A_2(1 - m'_{11})\right]\right\}$$

and solving for dr

$$dr = \frac{da}{E}\left[- p\, m'_{21} m'_{12} + q\, m'_{12} m'_{21} - v(1 - m'_{22})(1 - m'_{11}) + p\, m'_{21}(1 - m'_{22}) - q\, m'_{12}(1 - m'_{11}) + v\, m'_{12} m'_{21}\right]$$

$$dr = \frac{da}{E}\left\{ p\left[m'_{21}(1 - m'_{22}) - m'_{12} m'_{21}\right] + q\left[m'_{12} m'_{21} - m'_{12}(1 - m'_{11})\right] + v\left[m'_{12} m'_{21} - (1 - m'_{22})(1 - m'_{11})\right]\right\}.$$

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[30] Vanek, op. cit., p. 139, seems to have made an error in calculating this coefficient determinant, for he writes the sign of the second term as positive. His discussion of stability conditons and his worked-out example suffer because of this mistake.

We notice that this is the same as our expression for E, with A_1, A_2 and A_3 replaced by -p, -q, and v respectively.

As we saw earlier, different possibilities of autonomous increases in demand can be examined by assigning the appropriate values to p, q, and v. In the case of an increase in demand for exports or home produced goods, p, q, and v will always be either 1, -1, or 0. However, when we examine the effects of capital flows, this will not necessarily be the case, for a capital flow can be spent in three distinct ways, as mentioned in Chapter I, and in an infinite number of combinations of these three. Thus, p represents the change in the amount spent on country 1's output, q represents the change in the amount spent on country 2's output, and v represents the amount of capital which enters the foreign exchange market. Each of these is expressed as a fraction of the capital inflow. This definition of v may seem to be contradictory to what was said on page 44 of this section, for there we said that all net capital flows would be included in the "a" term of equation (7), and obviously all net capital flows need not enter the foreign exchange market. This apparent contradiction is perhaps best cleared up by an example. Suppose that one billion dollars is lent to country 1, and further suppose that this money is spent half on imports, and half on domestically-produced goods. Now this could be treated either by assuming that imports of country 1, m_{21}, increase by one-half billion and that capital flows (included in "a") increase by one billion, or by considering

the net change to be one-half billion,[31] i.e., by writing $v = \frac{1}{2}$. In fact, this latter is the only way with which such a situation can be dealt, for all autonomous changes in any of the variables must be included in the "a" term.

Before moving on to an investigation of some of the properties of our model, a brief discussion on the conditions necessary to insure that our model is stable would seem to be appropriate. The most satisfactory method for determining stability conditions is to investigate the dynamic properties of the model. While such an investigation is not within the scope of this thesis, Laursen and Metzler, in discussing a model identical to ours, except for the method of dealing with imports and exports, as mentioned above, have developed these dynamic stability conditions, and their results can be applied to our model.[32] They show that there are three conditions which must be fulfilled, and in discussing the third of these they say, "thus, one of the conditions of stability . . . is that . . . the determinant of our static system, must be positive."[33] They go on to say that:

> It can easily be shown . . . that . . . the third condition of stability, . . . is the critical one; if this condition is satisfied also; and if the third condition is not satisfied, the system is of course unstable regardless of whether the first two conditions of stability are satisfied or not.[34]

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[31] This is true, of course, only as long as $r = 1$.

[32] Laursen and Metzler, loc. cit., pp. 292-299.

[33] Ibid., p. 296.

[34] Ibid., p. 297.

Thus, assuming that this argument holds equally well for our model, and the similarity of the models indicates that it does, we can say that our model will be stable as long as the value of the coefficient determinant is positive. Equation (11) as it stands, however, is rather complicated, and it is impossible to tell by looking at it whether it will be positive or negative. It is thus desirable that it be simplified. It is shown in Appendix B, that under the assumption that the propensities in both countries are identical, E will be positive as long as

$$\frac{(1 - m'_{12} - m'_{22})}{m'_{12}} [(d_{12} + d_{21}) - 1] + d_{21}(1 - f_1) + d_{12}(1 - f_2) > 2 \text{ - - - - (14)}$$

where all terms are expressed as absolute values. The terms f_1 and f_2 show the degree of substitution associated with a particular change in the exchange rate. Their values will generally lie between zero and unity, depending on the degree of substitution between imports and domestic produce. If the degree of substitution is high, in other words, if an increase in imports is associated with almost as large a decrease in consumption of domestically-produced goods, then f will approach unity. If the degree of substitution is low, in other words, if an increase in imports is associated with almost no decrease in consumption of domestic goods, then f will be near zero. Equation (14) is seen to be a generalized Marshall-Lerner condition, for it contains the expression $(d_{12} + d_{21}) - 1$. The Marshall-Lerner condition, however, while necessary

for the stability of our model, is not sufficient, for $(d_{12} + d_{21})$ could be greater than unity while the whole expression remained less than 2.

Equation (14) can be simplified further by making assumptions about the values of f_1 and f_2. The limiting conditions, under normal circumstances, are $f_1 = f_2 = 1$, and $f_1 = f_2 = 0$, i.e., there is complete substitution in both countries, or there is no substitution in either country. Assuming first that $f_1 = f_2 = 1$, equation (14) reduces to

$$\frac{(1 - m'_{12} - m'_{22})}{m'_{12}} [(d_{12} + d_{21}) - 1] > 2$$

or

$$\frac{(1 - m'_{12} - m'_{22})}{2m'_{12}} [(d_{12} + d_{21}) - 1] > 1. \text{- - - - - - (15)}$$

The expression $(1 - m'_{12} - m'_{22})$ is the marginal propensity to save. From equation (15) we can see that the marginal propensity to save (mps) may have to be considerably larger than the marginal propensity to import (m'_{12}) if the expression is to be negative, even if $(d_{12} + d_{21})$ is larger than 1. In fact, even when $d_{12} = d_{21} = 1$, the mps must be twice as large as m'_{12} to insure stability, and, of course, the closer m'_{12} gets to the value of the mps, the larger the sum of the elasticities of demand for imports needs to be to insure stability. Thus, if the mps $= m'_{12}$, $(d_{12} + d_{21})$ must be greater than three. This may have implications for Canada, for it is quite conceivable that m'_{12} is larger than the mps. In Chapter III the

"best guess" for these two variables is .15 and .10 respectively. If these are correct, then $(d_{12} + d_{21})$ must be greater than 4 to insure stability of the model.

If $f_1 = f_2 = 0$, then equation (14) reduces to

$$\frac{(1 - m'_{12} - m'_{22})}{m'_{12}} [(d_{12} + d_{21}) - 1] + d_{21} + d_{12} - 2 > 0$$

or

$$\frac{(1 - m'_{12} - m'_{22})}{m'_{12}} [d_{12} + d_{21} - 1] + [d_{21} + d_{12} - 1] - 1 > 0$$

which becomes

$$[\frac{(1 - m'_{12} - m'_{22})}{m'_{12}} + 1][(d_{12} + d_{21}) - 1] > 1 - - - \quad (16)$$

or, alternatively,

$$\frac{(1 - m'_{22})}{m_{12}} [(d_{12} + d_{21}) - 1] > 1.- - - - - - - - - \quad (17)$$

The stability conditions here are not as stringent as in equation (15) and in fact, from examining equation (16), we see that a value of $(d_{12} + d_{21})$ equal to 2 is sufficient to insure stability, for the left-hand bracketed expression of the left-hand side of equation (16) must always be greater than 1. This, of course, assumes that both the marginal propensity to save and the marginal propensity to import are greater than zero. We assume that this will normally be the case.

The question of which of these two assumptions, i.e. $f_1 = f_2 = 0$, or $f_2 = f_1 = 1$, is most likely to be fulfilled does not seem to have any clear-cut answer. The former, that there will be no substitution, is certainly unlikely, but on the other hand it is very doubtful that

there will be complete substitution. Thus, the most we can say is that these two equations, (15) and (17), express the extreme conditions, and if we know approximate values for our propensities we can calculate the two sets of elasticities related to the stability condition. For example, if $m'_{12} = .05$, and $m_{22} = .85$, then by equation (15), $(d_{12} + d_{21})$ must be greater than two to insure stability. By equation (17), using these same values, $(d_{12} + d_{21})$ must be greater than 1 1/3 to guarantee stability. Thus, the stability of the model is assured if the sum of the elasticities is greater than 2, while at the same time, the model may be stable with a value of the sum of the elasticities as small as 1 1/3.

We must, however, always keep in mind Harberger's discussion of "the problem of translation" and "the fallacy of completeness".[35] As he has suggested, the stability conditions which we have derived depend on the assumptions we have made. Our main assumptions have been that pure Keynesian conditions exist and that both countries are identical. In analysing the economic interrelations of Canada and the United States, the first of these is certainly an oversimplification, and the second is clearly violated. Thus we cannot realistically conclude much from the above analysis, except perhaps that the value of the elasticity of demand for imports might need to be higher than indicated by the Marshall-Lerner condition, and that

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[35] Harberger, loc. cit., p. 857.

stability depends on the degree of substitution and on the marginal propensities of the two countries as well as on the elasticities of demand for imports.

We must also be careful to avoid the pitfall of assuming our model to be complete in any economic sense. Many factors have been ignored, some of which will undoubtably affect the stability conditions. Perhaps the most important of these is the monetary influence. Tsiang, for example, warns that stability conditions will be quite different if we drop the Keynesian assumption of a fixed interest rate.[36] He suggests that under more realistic monetary assumptions, the stability conditions would not be as stringent.

Our stability conditions thus must be viewed with a certain amount of scepticism. Throughout our analysis we will assume our model to be stable as long as E is larger than zero, but if E should, under certain conditions, become negative, we will not become too alarmed. We will now proceed with a closer examination of the model.

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[36] Tsiang, loc. cit., p. 935.

CHAPTER III

AN EXAMINATION OF THE MODEL

1. Introduction

This chapter is of three main parts. In the first, the model developed in Chapter II will be used to investigate the three types of capital inflows that Penner has defined, and which were discussed briefly in Chapter I. The second part will be devoted to a discussion of the effects on national income of different assumed values of the parameters, and the third part will be devoted to the calculation of values of the parameters which would result in no change in national income--the borderline conditions that were mentioned in Chapter I.

2. Penner's Three Types of Capital Inflow

Penner's type 1 capital inflow was "a capital inflow used to purchase newly-produced Canadian goods and services".[1] This means that in the general equation for changes in the income of country 1 developed in Chapter II, that is

$$dY_1 = \frac{da}{E}\left\{ p[A_2 m'_{12} - A_3(1 - m'_{22})] - qm'_{12}[A_1 + A_3] - v[A_2 m'_{12} + A_1(1 - m'_{22})]\right\} \quad -- (1)$$

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[1] Penner, loc. cit., p. 529.

p, which is the amount of the inflow spent on domestic output, expressed as a fraction of the total inflow, will be 1, and v, which is the amount of the inflow which enters the foreign exchange market, expressed as a fraction of total inflow, will also be 1. The value we should assign to q, the change in spending on the output of country 2, expressed as a fraction of the total capital inflow, cannot be so easily decided, for of such effects Penner had nothing to say. The two extreme conditions, of course, will be where q = 0, that is, where the transfer does not affect spending in country 2, implying that it is financed out of savings, idle balances, or by expansion of the money supply, and where q = -1, that is, where the transfer reduces spending on the production of country 2 by the exact amount of the transfer. Both these possibilities will be examined, and the changes of national income under these two assumptions will be compared. Before proceeding with this investigation, however, the relationship between two of the elasticities of our model will be discussed, for this relationship has a direct bearing on one of Penner's assumptions.

Penner says, after his analysis of the direct effects of his type 1 capital inflow, that "the crucial assumption in the above analysis is that total Canadian-financed demand for foreign plus domestic goods remains constant".[2] He also says, that if such a relationship

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[2] Ibid.

exists, then "the increase in imports will be matched exactly by a reduction in expenditures by Canadians on import-competing products".[3] This implies a direct relationship between d_{21}, country 1's elasticity of demand for imports, and d_{11}^{+}, the cross-elasticity of demand of country 1 for its own output with respect to the price of imports. From Chapter II,

$$d_{21} = \frac{\partial m_{21}}{\partial r} \frac{r}{m_{21}} \qquad \text{- - - - - - - - - - - - - - - - - -} \quad (2)$$

and

$$d_{11}^{+} = \frac{\partial m_{11}}{\partial r} \frac{r}{m_{11}} \qquad \text{- - - - - - - - - - - - - - - - - -} \quad (3)$$

Penner's assumption is that $\frac{\partial m_{21}}{\partial r} = \frac{\partial m_{11}}{\partial r}$, although, of course, they will be opposite in sign. From (2) we can write

$$r = \frac{\partial r}{\partial m_{21}} m_{21} d_{21}$$

and from (3)

$$r = \frac{\partial r}{\partial m_{11}} m_{11} d_{11}^{+} .$$

Since r must be the same for both, we can write

$$\frac{\partial r}{\partial m_{11}} m_{11} d_{11}^{+} = \frac{\partial r}{\partial m_{21}} m_{21} d_{21}$$

or

$$d_{11}^{+} = \frac{\partial m_{11}}{\partial r} \frac{\partial r}{\partial m_{21}} \frac{m_{21}}{m_{11}} d_{21}$$

or

$$d_{11}^{+} = f_1 \frac{m_{21}}{m_{11}} d_{21} \quad \text{where } f_1 = \frac{\partial m_{11}}{\partial r} \frac{\partial r}{\partial m_{21}} .$$

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[3] Ibid. A discussion of this assumption is found in Chapter I.

In a similar way we can write

$$d_{22}^{+} = f_2 \frac{m_{12}}{m_{22}} d_{12} \quad \text{where } f_2 = \frac{\partial m_{22}}{\partial r^*} \frac{\partial r^*}{\partial m_{12}} .$$

Penner's assumption implies that

$$d_{11}^{+} = \frac{m_{21}}{m_{11}} d_{21} .$$

It is very important, in our model, to realize that this relationship between the two elasticities exists.[4]

Penner has suggested that under his assumptions, the change in national income due directly to a type 1 capital inflow will be zero. The easiest way to investigate this contention is to set $dY_1 = 0$ and calculate what this implies about the parameters. In equation (1), since da/E is not equal to zero, dY_1 will be equal to zero if

$$p [A_2 m'_{12} - A_3(1 - m'_{22})] - q m'_{12} [A_1 + A_3] - v [A_2 m'_{12} + A_1(1 - m'_{22})] = 0.$$

We know that $p = 1$ and $v = 1$, and assuming first that $q = 0$, we can write

$$A_2 m'_{12} - A_3 (1 - m'_{22}) - A_2 m'_{12} - A_1(1 - m'_{22}) = 0$$

$$(1 - m'_{22}) [-A_3 - A_1] = 0 \quad - - - - - - - - - - - - - - - (4)$$

From Chapter II we know that

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[4]Vanek, op. cit., p. 144, does not seem to be aware of this relationship. For example, he calculates values for dY_1, dY_2, and dr, assuming that $d_{11}^{+} = d_{22}^{+} = .05$, and using three different values for $d_{12} = d_{21}$, namely, -1, -2, and -3. The values of f_1 for these three values of the elasticity of demand for imports are found to be -1.2, -.6, and -.4 respectively. The first of these is clearly impossible under any normal assumptions, and of the other two, the last seems somewhat improbable. Vanek's results and conclusions are, of course, distorted by his failure to appreciate the relationship which exists between these two elasticities.

$$A_1 = d_{11}^{+} m_{11} - d_{12} m_{12}$$

and

$$A_3 = m_{21} + d_{21} m_{21} + d_{12} m_{12}$$

therefore

$$(1 - m'_{22}) [- d_{11}^{+} m_{11} + d_{12} m_{12} - m_{21} - d_{21} m_{21} - d_{12} m_{12}] = 0$$
$$- (1 - m'_{22}) [m_{21} + d_{11}^{+} m_{11} + d_{21} m_{21}]$$

since $d_{11}^{+} = f_1 \frac{m_{21}}{m_{11}} d_{21}$

we can write

$$- (1 - m'_{22}) [m_{21} + f_1 \frac{m_{21}}{m_{11}} m_{11} d_{21} + d_{21} m_{21}] = 0$$

or

$$- (1 - m'_{22}) m_{21} d_{21} [\frac{1}{d_{21}} + f_1 + 1] = 0.$$

Changing to absolute values we have

$$(1 - m'_{22}) m_{21} d_{21} [(1 - \frac{1}{d_{21}}) - f_1] = 0 - - - - -(5)$$

Now, assuming that the marginal propensity to spend is less than 1, and that m_{21} is positive, it is obvious that dY_1 will be zero, only when

$$(1 - \frac{1}{d_{21}}) = f_1 - - - - - - - - - - - - - - - - - - -(6)$$

There are, of course any number of combinations of d_{21}, $\frac{\partial m_{11}}{\partial r}$, and $\frac{\partial m_{21}}{\partial r}$ which would make $dY_1 = 0$. However, if Penner's assumption holds, i.e. that $\frac{\partial m_{11}}{\partial r} = \frac{\partial m_{21}}{\partial r}$, then dY_1 will be zero only when d_{21} is infinite. This must be regarded as rather unlikely. Thus, when $q = 0$, dY_1 will be equal to zero only if equation (6) holds, which could only

occur by chance. If Penner's "crucial assumption" holds, i.e., that $f_1 = 0$, then dY_1 will be equal to zero only if country 1's demand for imports is perfectly price elastic.

We now assume that $q = -1$. Then, setting $dY_1 = 0$, equation (1) becomes

$$\cancel{A_2 m_{12}} - A_3(1 - m_{22}) + A_1 m_{12} + A_3 m_{12} - \cancel{A_2 m_{12}} - A_1(1 - m'_{22}) = 0$$

$$- (1 - m'_{22})[A_1 + A_3] + m'_{12}[A_1 + A_3] = 0$$

or

$$m'_{12} - (1 - m'_{22})[A_1 + A_3] = 0$$

then

$$- (1 - m'_{12} - m'_{22})[A_1 + A_3] = 0$$

or

$$(1 - m'_{12} - m'_{22})[-A_1 - A_3] = 0 \text{ - - - - - - - - - - (7)}$$

which is equation (4) with $(1 - m'_{22})$ replaced with $(1 - m'_{12} - m'_{22})$. We can thus write

$$(1 - m'_{12} - m'_{22})\, m_{21} d_{21} \left[\left(1 - \frac{1}{d_{21}}\right) - f_1\right] = 0. \text{ - - - (8)}$$

Then, assuming that $(1 - m'_{12} - m'_{22})$ (the marginal propensity to save), is positive, the conclusions reached for equasion (5) will hold for equation (8). The sign, then, of a change in the national income of country 1, is not affected by the value of q, although the magnitude certainly will be. Equations (5) and (8) can be easily converted into expressions for the actual change in national income by multiplying them by the factor da/E.

Equations (5) and (8) are, of course, useful for showing when dY_1 will be positive or negative as well as for showing when it will be zero. We can write the three

conditions:

$$dY_1 = 0 \text{ if } (1 - \frac{1}{d_{21}}) = f_1 \text{ - - - - - - - - - -(9)}$$

$$dY_1 > 0 \text{ if } (1 - \frac{1}{d_{21}}) > f_1 \text{ - - - - - - - - - -(10)}$$

$$dY_1 < 0 \text{ if } (1 - \frac{1}{d_{21}}) < f_1 \text{ - - - - - - - - - -(11)}$$

These will hold regardless of the effects of the capital inflow on the lending country.

One aspect of these three relationships is rather puzzling on first observation. Using equation (5) or (8), we are able to calculate the sign of a change in the real income of country 1, without any apparent consideration being given to exports. This is not evident from an examination of equation (1), the general expression of changes in income of country 1, for in the fully expanded form of this equation, d_{12} , the elasticity of demand for country 2's imports, appears no less than four times. All these cancel out, however, before equation (8), the most simplified form of the relationship, is derived. It can easily be seen, however, by examining the steps of our simplification process, that this would not be true if p and v were not equal. As soon as these differ by even a small amount, the term d_{12} shows up in the final expression. It is apparent then that it is the equality of p and v that cancels out the effect of exports. The question we must examine now is why this should be so.

Equations (5) and (7) in Chapter II both contain the expression for the exports of country 1, i.e., m_{12} $(Y_2 , \frac{1}{r})$.

We notice, however, that this term has a positive sign in equation (5) but a negative sign in equation (7). We would thus expect these terms to cancel out if p = v. From examining expression (11), remembering what the A's are, we see that this will be the case. We also observe that changing all the signs of equation (7) will not affect this conclusion. Thus from a mathematical point of view we would not expect d_{12} to appear in the numerator of our expression for the change in the income of country 1. From an economic point of view, however, the answer is not quite so simple, and depends, in the final analysis, on the assumptions of our model. Before going into this, however, it must be pointed out again that we are not arguing that d_{12} does not have an influence on dY_1, but only that d_{12} cannot affect the sign of changes in the income of country 1. In other words, d_{12} disappears from the numerator of equation (1), but not necessarily from the denominator. We are assuming here that E will be positive.

The economic reason why d_{12} cannot affect the sign of dY_1 depends on our assumption that domestic prices in both countries are constant. This assumption allowed us to conclude, on page 42 of Chapter II, that "differences in the price level will occur only with changes in the exchange rate." Thus we have directly linked price changes and exchange rate changes. From an examination of the equation for the change in the exchange rate, we see that the value of d_{12} is one of the factors which will determine how much the exchange rate must change to bring the

model into equilibrium after it is disturbed by some autonomous force. In section 3 of this chapter, it is shown that increasing the absolute value of d_{12} will increase the value of E, which, of course, will decrease the amount that the exchange rate changes. This means that prices, and consequently, country 2's imports, will not increase by as much as they would have had d_{12} not increased. The chain of events is thus as follows. If d_{12} increases, country 2's imports will increase. This increase in d_{12}, however, decreases the amount by which the exchange rate must change to re-establish equilibrium. This decrease in the price change of country 2's imports, will decrease the amount by which imports increase. Thus when $p = v$, the two effects on country 2's imports cancel each other out, and d_{12} disappears from the numerator of equation (1).

We have shown, then, that Penner's contention that capital inflows of type 1 will have no effect on national income is certainly not a necessary condition, and, in fact, is rather unlikely from a theoretical point of view. The probability is much higher that the capital inflow will either decrease or increase national income. We have also seen that Penner's crucial assumption, that is, that the increase in imports will be exactly equal to the decrease in spending on import-competing products, will lead to a fall in national income under any normal assumption about the elasticity of demand for imports. We will now turn to an examination of Penner's second type of capital inflow.

Penner deals with his second type of capital inflow, "a capital inflow used to finance purchases of foreign goods",[5] in less than a third of a page. Our discussion will be almost equally as brief. Penner's argument is, that when a capital inflow is used to purchase foreign goods, it does not increase demand for home produce, and it does not affect the exchange rate. Thus, he concludes that such a capital inflow will have no direct influence on the level of domestic output. In equation (1), p and v will both be equal to zero, and the only term which will not disappear will be the one containing q. As we remember, q is the amount of the inflow used to increase the spending on country 2's output. Again there are two extreme values which q could assume. If the initial capital inflow reduces spending on country 2's output by q, then the increase in spending by amount q will have no net effect, and equation (1) will be equal to zero. If, on the other hand, the initial capital inflow was financed, for example, from country 2's savings, then q will be equal to 1.[6] If q = 1, then equation (1) becomes

$$dY_1 = \frac{da}{E} [- m'_{12} (A_1 + A_3)].$$

Setting dY_1 equal to zero, we have

$$m'_{12} [-A_1 - A_3] = 0 \text{ - - - - - - - - - - - - - - - - (12)}$$

Equation (12) is similar to equation (4) and we can thus

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[5]Penner, loc. cit., p. 532.

[6]Penner's argument that the exchange rate will not be affected assumes that q = 0.

write

$$m'_{12}\, m_{21}\, d_{21} \left[\left(1 - \frac{1}{d_{21}}\right) - f_1 \right] = 0 \quad \text{- - - - - - - - -(13)}$$

and, of course, the same conclusions as before follow.

Thus, we can conclude that if the net effect on country 2 of this type of capital inflow is zero, then the effect on country 1 will be zero, but if the effect on the national income of country 2 is positive, then the national income of country 1 could decrease, increase, or remain the same, depending on the values of d_{21}, and f_1. We also note that if $\left(1 - \frac{1}{d_{21}}\right)$ is larger than f_1, any positive value of q, no matter how small, will guarantee that the national income of country 1 will increase.

Before going on to a discussion of Penner's third type of capital inflow, the relationships among equations (5), (8) and (13) will be examined briefly. If we assume values for d_{21} and f_1 such that $\left(1 - \frac{1}{d_{21}}\right)$ is larger than f_1, then all three will be positive as long as $q \neq 0$. Assuming that the marginal propensity to import and the marginal propensity to spend are both positive, equation (5) will always have the greatest effect on national income, for $(1 - m'_{22})$ must be larger than either $(1 - m'_{12} - m'_{22})$ or m'_{12}. If the marginal propensity to save is larger than the marginal propensity to spend, then equation (8) will be larger than equation (13). While it is by no means certain that the mps will always be larger than m'_{12}, this would seem to be the normal case, especially if we consider country 2 to be the United States,

or "the rest of the world". The relative sizes of the effects of these three equations are, of course, what we would expect. We intuitively feel that a capital inflow spent on domestic goods would have a larger effect on Canada than if the inflow were spent on foreign goods. Of course, if the bracketed expression is negative, then a capital inflow spent on domestic goods will have a larger contractionary effect on the Canadian economy than if it had been spent on foreign produce. Under such circumstances, then, an inflow should be spent on imports rather than domestically-produced goods, if the contractionary effects are to be minimized.

This is perhaps an appropriate time to point out that the same type of analysis as developed above can be used to show the effects of an autonomous increase in exports, and in fact, equation (5) would be used to examine the change in national income due to changes in exports. Thus, if increased exports raise national income, so will capital inflows, whether spent on newly-produced Canadian goods or on newly-produced foreign goods, assuming, of course, that this latter represents a net addition to foreign demand, i.e., $q \neq 0$. This does not seem to be generally recognized. It is usually assumed that exports increase national income while capital inflows decrease national income. While either of these may, in fact, be true, they cannot, from a theoretical point of view, both be true at the same time.

Penner's third type of capital inflow is "a capital inflow used to finance purchases of already existing real or financial assets in Canada".[7] He argues that since this type of inflow appreciated the exchange rate without increasing the demand for Canadian-produced goods, it is more liable to be deflationary than the other two. In our model, p would be zero, v would be 1, and q could take on any value between 0 and -1, both inclusive. If q = 0, then equation (1) becomes

$$dY_1 = \frac{da}{E} [-m'_{12} A_2 - (1 - m'_{22}) A_1] \text{.- - - - - - - -(14)}$$

Setting equation (14) equal to zero, and remembering that $(1 - m'_{22}) = m'_{12} + mps$, we can write

$$- m'_{12} A_2 - m'_{12} A_1 - mps\ A_1 = 0$$

$$- m'_{12} [A_2 + A_1] - mps\ A_1 = 0$$

$$- m'_{12} [d_{21} m_{21} - d^+_{22} m_{22} + d^+_{11} m_{11} - d_{12} m_{12}]$$

$$- mps [d^+_{11} m_{11} - d_{12} m_{12}] = 0.$$

Now, since

$$d^+_{11} = f_1 \frac{m_{21}}{m_{11}} d_{21}$$

and

$$d^+_{22} = f_2 \frac{m_{12}}{m_{22}} d_{12}$$

we can write

$$- m'_{12} [d_{21} m_{21} - f_2 m_{12} d_{12} + f_1 m_{21} d_{21} - d_{12} m_{12}]$$

$$- mps [f_1 m_{21} d_{21} - d_{12} m_{12}] = 0$$

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[7] Ibid.

now, assuming that $m_{21} = m_{12}$

$$- m_{12}m_{12}[\, d_{21} - f_2 d_{12} + f_1 d_{21} - d_{12}\,] - mps\, m_{12}[\, f_1 d_{21} - d_{12}\,] = 0$$

collecting terms

$$- m'_{12}\,[\, d_{21}(1 - f_1) - d_{12}(1 + f_2)] - mps\,[\, d_{21} f_1 - d_{12}\,] = 0$$

in absolute values

$$m'_{12}\,[\, d_{21}(1 - f_1) - d_{12}(1 - f_2)] - mps\,[\, d_{12} + d_{21} f_1] = 0$$

expanding, we have

$$m'_{12}\, d_{21}(1 - f_1) - m'_{12} d_{12}(1 - f_2) - mps\, d_{12} - mps\, d_{21} f_1 = 0$$

collecting terms

$$d_{21}\,[\, m'_{12}(1 - f_1) - mps\, f_1] - [\, d_{12} m_{12}(1 - f_2) + mps\,] = 0 - - - \quad (15)$$

Now, if there is complete substitution in both countries, i.e., if $f_1 = f_2 = 1$, then we will have

$$d_{21}\,[\, -mps] - d_{12}\,[mps] = 0 - - - - - - - - - - - - - \quad (16)$$

and dY_1 must be negative. If there is no substitution in either country, i.e., if $f_1 = f_2 = 0$, then

$$d_{21}[m'_{12}\,] - d_{12}[m'_{12} + mps] = 0$$

or

$$d_{21} - d_{12}\,[\, 1 + \frac{mps}{m'_{12}}\,] = 0.- - - - - - - - - - - - - - \quad (17)$$

Whether the left-hand expression is negative, positive, or zero, depends on the value of mps/m'_{12}, d_{21}, and d_{12}. In any case, d_{21} will have to be larger, and probably considerably larger, than d_{12} if dY_1 is to be positive. For

example, if the mps is twice as large as m'_{12}, d_{21} must be more than three times as large as d_{12} for dY_1 to be positive. If $f_1 = 1$, and $f_2 = 0$, then (15) becomes

$$d_{21}[-mps] - d_{12}[m'_{12} + mps] = 0 \quad \text{- - - - - - - (18)}$$

and dY_1 must be negative. If $f_1 = 0$ and $f_2 = 1$, then (15) becomes

$$d_{21} m_{12} - d_{12} mps = 0$$

or

$$d_{21} - d_{12} \frac{mps}{m'_{12}} = 0 \quad \text{- - - - - - - - - - - - - - - - (19)}$$

Similar to equation (17), equation (19) could be negative, positive or zero, depending again on the values of d_{21}, d_{12} and mps/m'_{12}.

Capital flows of type 3, then, when $q = 0$, cannot definitely be said to have a positive or negative effect on national income. It would seem, however, that there is a strong possibility that the effect will be negative. For example, it seems unlikely that either f_1 or f_2 will be less than .5, and if they are both .5, then equation (15) becomes

$$d_{21}[m'_{12} - mps] - d_{12}[m'_{12} + 2mps] = 0. \text{- - - - -(20)}$$

Now if the mps is larger than m'_{12}, which will probably be the case, then dY_1 must be negative. Even if m'_{12} is larger than the mps, dY_1 will be negative unless d_{21} is much larger than d_{12}, for $[m'_{12} + 2mps]$ will certainly be much larger than $[m'_{12} - mps]$. Thus it would seem safe to conclude that when $q = 0$, a type 3 capital inflow will normally have a contractionary effect on national income.

If $q = -1$, then equation (1) becomes

$$dY_1 = \frac{da}{E}\left\{m'_{12}[\,A_1 + A_3\,] - m'_{12}\,A_2 - (1 - m'_{22})\,A_1\right\} \text{---(21)}$$

Setting $dY_1 = 0$, and writing $(1 - m'_{22})$ as $m'_{12} + mps$, we can write

$$m'_{12}\,A_1 + m'_{12}\,A_3 - m'_{12}\,A_2 - m_{12}\,A_1 - mps\,A_1 = 0$$

collecting terms

$$m'_{12}\,[\,A_3 - A_2\,] - mps\,A_1 = 0$$

$$m'_{12}\,[\,m_{21} + d_{21}\,m_{21} + d_{12}\,m_{12} - d_{21}\,m_{21} + d^{+}_{22}\,m_{22}\,] - mps\,[\,d^{+}_{11}\,m_{11} - d_{12}\,m_{12}] = 0.$$

Remembering that

$$d^{+}_{11} = f_1\,\frac{m_{21}}{m_{11}}\,d_{21}$$

and

$$d^{+}_{22} = f_2\,\frac{m_{12}}{m_{22}}\,d_{12}$$

we can write

$$m'_{12}[m_{21} + d_{12}\,m_{12} + f_2\,m_{12}\,d_{12}] - mps[f_1 m_{21}\,d_{21} - d_{12}\,m_{12}] = 0.$$

Now, assuming $m_{21} = m_{12}$, we have

$$m'_{12}[1 + d_{12} + f_2\,d_{12}] - mps[f_1\,d_{21} - d_{12}] = 0$$

writing in absolute values

$$-\,m_{12}\,d_{12}\left[\left(1 - \frac{1}{d_{12}}\right) - f_2\right] - mps[d_{12} + f_1 d_{21}] = 0 \text{---(22)}^{8}$$

Assuming that $f_2 = f_1 = 1$, equation (22) becomes

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[8] This equation is rather interesting, because the left-hand bracketed expression is the same as the one derived for analysing Penner's type 1 capital flow, except, of course, that it refers to country 2 rather than to country 1. This expression tells us that if an appreciation of the exchange rate (a depreciation of country 2's currency) lowers national income in country 2, it will tend to lower income in country 1. This is, of course, what we would expect.

$$- m'_{12}\, d_{12}\left[\, 1 - \frac{1}{d_{12}} - 1\right] - mps\,[d_{12} + d_{21}] = 0$$

or

$$m'_{12} - mps\,[d_{12} + d_{21}] = 0. \qquad (23)$$

This expression will be negative with any normal values of the elasticities of demand for imports. If $f_2 = f_1 = 0$, equation (22) becomes

$$- m'_{12}\, d_{12}\left[\, 1 - \frac{1}{d_{12}}\,\right] - mps\,[\, d_{12}\,] = 0$$

or

$$- m_{12}\,[d_{12} - 1] - mps\,[\, d_{12}\,] = 0. \qquad (24)$$

This expression will be negative as long as d_{12} is greater than zero.

We can thus conclude that a type 3 capital inflow will normally be contractionary, regardless of whether $q = 0$ or -1. As we would expect, there is more chance of this type of capital inflow not being contractionary if $q = 0$.

We have now completed our analysis of Penner's three types of capital inflow. Studying each in isolation we have shown the conditions under which each would be expansionary or contractionary. We have seen that the possibility of a capital inflow being expansionary decreases as we move from type 1, through type 2, to type 3. We know, of course, that no capital flow will fall completely in any one of these classes, and in reality, combinations of these three would have to be considered. It is not convenient, however, to deal with such combinations in any

general sense, and we therefore leave such analyses to sections 3 and 4 of this chapter.

3. The Effect on National Income of Different Assumed Values of the Parameters

This section is an attempt to show how different assumptions about the values of our parameters will affect the calculated changes in national income. Only the national income of country 1 will be considered, for while a study of the influences of changes in the parameters on country 2's income and the exchange rate would be interesting, such an investigation is not within the scope of this thesis. Of course, if the two countries are identical, any conclusions derived for one will apply to the other.

A number of problems are encountered in this investigation. First of all, our results will depend on how we assume the capital is spent. It would be desirable, then, to consider each of Penner's three types of capital inflow. However, because of the relatively large number of parameters in our system, which makes calculations laborious, (there are, for example, twelve parameters in the expression for E), and because, as we shall see later, our results are not very conclusive, our time would not be well spent in examining all the possibilities. Our examination then will be limited to Penner's type 1 capital inflow.

Another problem that arises is the difficulty in

comparing changes in the different parameters, for if we are to compare the effects of these changes on national income, our assumed changes must in some way be comparable. The most satisfactory method is to assume constant percentage changes, although even this has disadvantages, for the results will depend on what we assume the original values to be. We would not expect, for example, that a certain percentage change in a propensity assumed to have a value of .2 would have the same effect on national income as the same percentage change if the value had been assumed to be .05. This problem is made more serious by the fact that our assumed values for these parameters cannot be considered to be much more than random choices.

We will call Canada country 1, and the United States country 2, and we will assume that these two countries trade only with each other. This assumption is not at all critical to our analysis, and is included only so that the propensities of both countries will add to unity. We will "guess" that the values of the parameters are as follows:

$m'_{11} = .70$	$m_{11} = 30$	$d^{+}_{11} = .30$
$m'_{21} = .20$	$m_{21} = 4$	$d_{21} = -4$
$m'_{22} = .85$	$m_{22} = 480$	$d^{+}_{22} = .05$
$m'_{12} = .05$	$m_{12} = 4$	$d_{12} = -4.$

It must be stressed that these values have been chosen more because they are easy to work with than because they represent approximations of the "true" values.

For a type 1 capital inflow where q = 0, the general

expression for the change in the income of country 1 reduces to

$$dY_1 = \frac{da}{E}(1 - m'_{22})[-A_1 - A_3]$$

where

$$A_1 = d^+_{11} m_{11} - d_{12} m_{12}$$

$$A_2 = d_{21} m_{21} - d^+_{22} m_{22}$$

$$A_3 = m_{21} + d_{21} m_{21} + d_{12} m_{12}$$

and where

$$E = A_1[m'_{12} m'_{21} - m'_{21}(1 - m'_{22})] - A_2[m'_{21} m'_{12} - m'_{12}(1-m'_{11})] - A_3[(1 - m'_{22})(1 - m'_{11}) - m'_{21} m'_{12}]$$

using the above values of the parameters, $A_1 = 25$, $A_2 = -40$, and $A_3 = -28$ and

$$E = 25(.01 - .03) + 40(.01 - .015) + 28(.045 - .01)$$

$$E = .28$$

therefore, assuming da = 1

$$dY_1 = \frac{(1 - m'_{22})}{E}(-A_1 - A_3)$$

$$= \frac{.15(3)}{.28}$$

$$dY_1 = 1.607142.$$

If we now assume d_{12} to be 25% less in absolute value, i.e., $d_{12} = -3$, then $A_1 = 21$ and $A_3 = -24$, A of course, remains the same. E now becomes:

$$E = 21(-.02) + 40(-.005) + 24(.035)$$

$$E = .22$$

and

$$dY_1 = \frac{.15(3)}{.22}$$

$dY_1 = 2.045454.$

The change in dY_1 is thus 2.045454 - 1.607142 = .438312 which is an increase of 27.27%.

If, however, we had assumed a 25% increase in the absolute value of d_{12}, i.e., $d_{12} = -5$, then $A_1 = 29$ and $A_3 = -32$, and

$$E = 29\ (-.02) + 40\ (-.005) + 32\ (.035)$$

$$E = .34$$

from which

$$dY_1 = \frac{(.15)3}{.34}$$

$$dY_1 = 1.323529$$

which represents a decrease in dY_1 of 17.65%. Thus we can say that in general, the smaller the absolute value of d_{12}, the larger will be the change in Y_1.

It will be interesting to find out how much our results would have differed if we had assumed other parameter values. If, for example, we had assumed d^{+}_{11} to be equal to .1 instead of .3, then $A_1 = 19$ and $E = .40$, so that

$$dY_1 = \frac{(.15)(9)}{.40}$$

$$dY_1 = 3.375.$$

Now, if $d_{12} = -3$, then $A_1 = 15$ and $A_3 = -24$, and

$$E = 15\ (-.02) + 40\ (-.005) + 24\ (.035)$$

$$E = .34$$

and

$$dY_1 = \frac{(.15)(9)}{.34}$$

$dY_1 = 3.970588$

which represents an increase of 17.65% in dY_1 as compared to the 27.27% increase calculated under the assumption that $d_{11}^{+} = .30$. To a considerable extent, then, our calculated changes depend on the values of the other parameters.

We will now investigate the effect of assuming different values of d_{21}. If d_{21} decreased in absolute value by 25%, i.e., $d_{21} = -3$, then; $A_2 = -36$ and $A_3 = -24$, and

$$E = 25\ (-.02) + 36\ (-.005) + 24\ (.035)$$

$$E = .16$$

$$dY_1 = \frac{.15(-1)}{.16}$$

$$dY_1 = -.937500.$$

This represents a decrease of 158.33% in dY_1. If we assume that $d_{21} = -5$, then $A_2 = -44$ and $A_3 = -32$, and

$$E = 25\ (-.02) + 44\ (-.005) + 32\ (.035)$$

$$E = .40$$

$$dY_1 = \frac{.15(7)}{.40}$$

$$dY_1 = 2.625000.$$

This represents an increase in dY_1 of 63.33%. If we had assumed initially that $d_{11}^{+} = .1$, then we would have found that changing d_{21} to -5 would have increased dY_1 by 25.07%.

From the analysis to this point we can conclude that the increase in national income will be larger, the more inelastic is the demand curve for imports in the borrowing country, and the more elastic is the demand curve in the

lending country. The first of these is what we would expect, and tells us only that national income will be more, the less a decrease in import prices causes the quantity of imports to increase. The second is not as easily explained, however, and deserves further consideration.

From an examination of our calculations, we notice that the change in national income resulting from the change in d_{12} is due totally to the change in E, for while A_1 and A_3 change in absolute value, the difference between them remains the same regardless of the value of d_{12}. Thus, as long as E is positive, the value of the elasticity of demand for imports in country 2 cannot affect the sign of the change in income in country 1. This is the same conclusion as we reached in section 1 of this chapter where we discussed Penner's type 1 capital inflow. We also notice that this is not the case for d_{21}, for here, the change in Y_1 is due to the change in the difference between A_1 and A_3, and, in fact, the changes due to changes in E are of opposite direction to those due to changes in the A's. We will now examine the coefficient determinant to find out if a decrease in the absolute value of d_{12} could cause a reduction in national income.

Examining the expression for E, we note that the first two terms must be negative and the last must be positive if the marginal propensity to save in both countries is positive, i.e., if $(1 - m'_{11}) > m_{21}$, and if $(1 - m'_{22}) > m_{12}$. Also, since A_2 does not change, whether the change in E is negative or positive will depend on the

first and third terms. The change in E will be negative if the change in the first term is larger than the change in the third, and positive if the change in the third is larger than the change in the first. However, as noted above, A_1 and A_3 change by the same absolute amount and thus the sign of the change in E depends on the difference in the absolute values of the bracketed expressions and not in the values of the A's. This means that the critical factors in the determination of the change in E, and thus the change in Y_1, are the marginal propensities, and not the elasticities, i.e., income changes, nor price changes. The value of E will increase with a decrease in the absolute value of d_{12} only if the absolute value of $[m'_{12}\, m'_{21} - m'_{21}\,(1 - m'_{22})]$ is larger than the absolute value of $[(1 - m'_{22})(1 - m'_{11}) - m'_{21}\, m'_{12}]$. Examining these expressions we see that this can be the case only if m'_{21} is larger than $(1 - m'_{11})$, i.e., if the marginal propensity to save is negative. We cannot, at this stage of our analysis, consider such a possibility, for up to this point we have assumed the mps to be positive. Dropping this assumption would force us to re-examine all our previous work. We conclude then that a decrease in the absolute value of d_{12} will increase dY_1 if the mps of country 1 is greater than zero. If the mps is equal to zero, changes in d_{12} will not affect dY_1 at all. The economic implications of this are simply that the smaller the marginal propensity to import of country 1, the more will income

increase, assuming that income does increase when the exchange rate appreciates.

Because of the fact that changes in d_{12} only affect the denominator of the expression for changes in the income of country 1, changes in d_{12} will not have as large an effect on dY_1 as will changes in d_{21}. This conclusion is verified by the calculations of the first part of this section.

Before going on to a discussion of the remaining parameters, it must be pointed out that the above calculations are based on one set of assumed values for the twelve parameters. These values were used because they illustrate the arguments put forth above, and because they are easily manipulated. They are, however, rather unreasonable. For example, they imply a marginal propensity to import of 5% with an average propensity to import of less than 1%, and a value of f_2 considerably greater than unity; both highly unlikely situations. It can be shown, however, that changing these parameters does not affect our conclusions qualitatively, for if we assume that $m'_{22} = .88$, $m'_{12} = .02$, $m_{22} = 400$ and $d^+_{22} = .02$, assumptions which avoid the difficulties pointed out above, we find that if d_{12} changes to -3, dY_1 increases by 16.00%, and if d changes to -5, dY increases by 73.50%. These changes are in the same direction and of the same relative size, as those calculated using the other set of parameters.

Considering now the other parameters of the model, if we increase d^+_{11} by 25%, i.e., to .375, we have $A_1 = 27.25$,

which along with $A_2 = -40$ and $A_3 = -28$, gives us a value of $E = .235$. Thus

$$dY_1 = \frac{.15(.75)}{.235}$$

$$dY_1 = .478723.$$

This represents a decrease of 70.21% in dY_1. Assuming that d_{11}^+ decreases to .225, $A_1 = 22.75$ and $E = .325$. Thus

$$dY_1 = \frac{.15(5.25)}{.325}$$

$$dY_1 = 2.423077.$$

This represents an increase of 50.77% in dY_1.

If d_{22}^+ is reduced by 25% to .0375, A_2 becomes -34, and $E = .31$. Then

$$dY_1 = \frac{.15(3)}{.31}$$

$$dY_1 = 1.451613$$

which represents a decrease of 9.68% in dY_1. If d_{22}^+ increases by 25% to .0625, A_2 becomes -46, and $E = .25$. Thus

$$dY_1 = \frac{.15(3)}{.25}$$

$$dY_1 = 1.800$$

which represents an increase of 12.00% in dY_1.

These are the results we would expect, for an increase in d_{11}^+ implies a decrease in m_{11} which will decrease Y_1, and an increase in d_{22}^+ will increase m_{22}, which in turn will increase Y_2, and consequently, Y_1. Also, as we would expect, d_{11}^+ affects the income of country 1 much more than does d_{22}^+.

When the effects of changes in the propensities are considered, we must remember that the sum of the propensities for each country must add to one. Thus if we change one propensity we must change one or both of the other two. We will first of all assume that m'_{21} decreases by 25% to .15, and that this decrease is compensated for by an increase in the mps to .15. Thus m'_{11} remains the same. Under these conditions, and assuming our initial parameters, giving $A_1 = 25$, $A_2 = -40$ and $A_3 = -28$, we have

$$E = 25(.0075 - .0225) + 40(.0075 - .015) + 28(.045 - .0075)$$

$$E = .375$$

therefore,

$$dY_1 = \frac{.15(3)}{.375}$$

$$dY_1 = 1.200$$

which represents a decrease of 25.3% in dY_1. Now if we assume that in conjunction with the change in m'_{21}, m'_{11} changes to .75 and the mps remains the same, we have

$$E = 25(.0075 - .0225) + 40(.0075 - .0125) + 28(.0375 - .0075)$$

$$E = .265.$$

Therefore,

$$dY_1 = \frac{.15(3)}{.265}$$

$$dY_1 = 1.698113$$

which represents an increase of 5.66% in dY_1. Assuming that m'_{11} changes to .75 and that m'_{21} remains constant,

then

$$E = 25(.01 - .03) + 40(.01 - .0125) + 28(.0375 - .01)$$

$$E = .17$$

and

$$dY_1 = \frac{.15(3)}{.17}$$

$$dY_1 = 2.647058$$

which represents an increase of 64.71% in dY_1. Before drawing conclusions from this analysis we should perhaps examine what would happen under a different set of values of the A's. For example, if d_{21} has a value of -5 instead of -4, then we would have $A_1 = 25$, $A_2 = -44$, $A_3 = -32$, and for the first set of parameters we assumed, $dY_1 = 2.625$. Now, for $m'_{21} = .15$ with m'_{11} constant,

$$E = 25(-.015) + 44(.0075) + 32(.0375)$$

$$E = .495$$

and

$$dY_1 = \frac{.15(7)}{.495}$$

$$dY_1 = 2.121212$$

which represents a decrease of 19.19% in dY_1. For $m'_{21} = .15$, $m'_{11} = .75$ and the mps constant, we have

$$E = 25(-.015) + 44(.0050) + 32(.03)$$

$$E = .365$$

and

$$dY_1 = \frac{.15(7)}{.365}$$

$$dY_1 = 2.876712.$$

This represents an increase of 9.59% in dY_1. Now if

$m'_{11} = .75$ and m'_{21} is constant,

$$E = 25(-.02) + 44(.0025) + 32(.0275)$$

$$E = .27.$$

Thus

$$dY_1 = \frac{.15(7)}{.27}$$

$$dY_1 = 3.888$$

which represents a 48.15% increase in dY_1. These values show the same type of changes in dY_1 as those calculated using the other set of parameters. They are quantitatively different, but qualitatively the same.

These results seem quite reasonable, for when m'_{11} increases, m'_{21} remaining constant, we would expect dY_1 to increase. If, when m'_{11} increases, m'_{21} decreases (which will lower country 2's nationsl income), it seems reasonable that the increase in dY_1 will not be as large as when m'_{21} did not change, for the decrease in dY_2 will tend to decrease dY_1. When m'_{21} decreases and m'_{11} remains constant, the domestic multiplier remains the same, while the income of country 2 is decreased. Thus we would expect a net decrease in the income of country 1.

Turning now to country 2's propensities, if m'_{12} decreases to .0325, m'_{22} remaining the same and the mps increasing, we have

$$E = 25(.0075 - .030) + 40(.0075 - .01125) + 28(.045 - .0075)$$

$$E = .3375$$

and

$$dY_1 = \frac{.15(3)}{.3375}$$

$$dY_1 = 1.333$$

which represents a decrease of 17.11% in dY_1. If m'_{12} decreases to .0325 while at the same time m'_{22} increases to .8625, the mps remaining constant, we have

$$E = 25(.0075 - .0275) + 40(.0075 - .01125) + 28(.04125 - .0075)$$

$$E = .295$$

thus

$$dY_1 = \frac{.1375(3)}{.295}$$

$$dY_1 = 1.398305$$

which represents a decrease of 13.01% in dY_1. If m'_{12} changes to .8625, m'_{22} remaining constant and the mps decreasing, we have

$$E = 25(.01 - .0275) + 40(.01 - .015) + 28(.04125 - .01)$$

$$E = .2375$$

and

$$dY_1 = \frac{.1375(3)}{.2375}$$

$$dY_1 = 1.736421$$

which represents an increase in dY_1 of 8.04%.

These results are also what we would intuitively expect, for if m'_{12} decreases, m'_{22} remaining constant, it seems reasonable that dY_1 would decrease. The positive effect on dY_1 of an increase in m'_{22} will tend to counteract the negative effect on dY_1 due to the decrease in m'_{12}, and when m'_{22} increases, while m'_{12} remains constant, we would

expect dY_1 to increase. We also notice that changes in dY_1 due to changing the propensities of country 2 are less than those due to changing the propensities of country 1. This also seems intuitively reasonable.

The results of this section are not very startling and tend to confirm what we would have intuitively expected. There are so many parameters, and the effects of each of these depend so much on the initial values we assume, and because we have no exact information as to what the "true" values of these parameters are, we can say very little about which of them have the most effect on national income. We can, perhaps, reach a few tentative conclusions. It seems certain, for example, that the propensities and elasticities of country 1 have more effect than those of country 2. This is particularly true for the elasticity of demand for imports. We have verified our postulate of section 2 of this chapter that the elasticity of demand for imports of country 2 cannot affect the sign of changes in the national income of country 1, and we have further shown that any effect this elasticity will have will depend on the propensities of country 1. We have seen that the marginal propensities to import of both countries play a more important role than we might have expected, and we have found the cross elasticities of demand to be very crucial parameters, not only for the determination of the size of the change in national income, but also, as we saw in section 2, for the determination of

the sign as well. This applies more to d_{11}^{+} than to d_{22}^{+} as we would expect.

We must, of course, keep in mind the fact that our analysis has been concerned only with Penner's type 1 capital flow, and then only when q = 0. Our conclusions would probably be somewhat different for the other two types of capital inflows. The uncertainty of our results to this point, however, does not seem to justify further pursual of this analysis. We will thus go on to the final section of this chapter.

4. The Borderline Conditions

By borderline conditions we simply mean values of the parameters which would make our equation for the change in national income of country 1 equal to zero. There are, of course, an infinite number of combinations of parameter values which would result in such a situation, and only a few of these can be considered. We have also, in the second section of this chapter, derived general expression for each of the three types of capital inflows, from which the conditions necessary to insure that dY_1 be greater than zero are quite easily seen. We could not, however, derive a simple general formula which included all three types of possible capital use. It would be informative to gain some idea of what the parameters would need to be to insure that capital flows would be expansionary when the inflow is divided among the three possible uses. This section, then, is simply a number of

examples, all of which are of rather questionable significance because of the uncertainty which surrounds our guesses of the parameter values.

From Chapter II, our expression for the change in the income of country 1 is

$$dY_1 = \frac{da}{E}\Big\{ p[A_2 m'_{12} - A_3(1 - m'_{22})] - qm'_{12}[A_1 + A_3] - v[A_2 m'_{12} + A_1(1 - m'_{22})]\Big\} \text{ - - - - - - (1)}$$

where

$$E = A_1[m'_{12} m'_{21} - m'_{21}(1 - m'_{22})] - A_2[m'_{12} m'_{21} - m'_{12}(1 - m'_{11})] - A_3[(1 - m'_{22})(1 - m'_{11}) - m'_{12} m'_{21}] \text{ - - - - (2)}$$

and where

$$A_1 = d^+_{11}\ m_{11} - d_{12}\ m_{12} \text{ - - - - - - - - - - - - - - - - - (3)}$$

$$A_2 = d_{21}\ m_{21} - d^+_{22}\ m_{22} \text{ - - - - - - - - - - - - - - - - - (4)}$$

$$A_3 = m_{21} + d_{12}\ m_{12} + d_{21}\ m_{21} \text{ - - - - - - - - - - - - - (5)}$$

Four of the above fifteen parameters, namely m_{11}, m_{12}, m_{22}, and m_{21} can be given fairly accurate values, for they are directly measurable, and they are known for any time period in the past. Also, because we know from Chapter II that

$$d^+_{11} = f_1 \frac{m_{21}}{m_{11}} d_{21} \text{ - - - - - - - - - - - - - - - - - - - (6)}$$

we can see that m_{11} will disappear from the first term of the expression for A_1, as, of course, will m_{22} in the second term of the expression for A_2. Thus these values are not at all critical. We will assume m_{11} = 30 billion and m_{22} = 400 billion. The terms m_{21} and m_{12} must be equal, and we will assume that $m_{21} = m_{12}$ = 4 billion, a figure

reasonably close to the present level of Canadian exports. The values for the remaining parameters found in the A's, are, unfortunately, much more uncertain. The value of the elasticity of demand for imports has long troubled economists, and while many attempts have been made to estimate the value of this parameter, no values have been calculated which enjoy anything approaching universal acceptance. The values of the cross elasticities depend, from equation (6), on the degree of substitution, and on the elasticity of demand for imports. As the elasticities are perhaps our most uncertain parameters, we will leave them as the "variables" in our calculations.

Our marginal propensities we can guess at and perhaps not be too far wrong. We would expect, for example, that the marginal propensity to import will be somewhat higher than the average propensity, which we know from the values of imports and national income. The average propensity for country 2 is approximately .01, and thus the assumption that $m'_{12} = .02$ will in all likelihood be a high, rather than a low, guess. For Canada (country 1), the average propensity is approximately .12 and thus a guess of .20 is more likely high than low. The marginal propensity to save is often assumed to be about .1. Tentatively accepting this we have $m'_{11} = .7$ and $m'_{22} = .88$.

Before "guessing at" the values of our last three parameters, p, q and v, we must have clearly in mind what each of these is. The term p is the increase in spending

on newly produced Canadian goods or services, resulting directly from the capital inflow;[9] q is the change in spending on the goods and services of country 2 resulting directly from the capital inflow; and v is the amount of capital which enters the foreign exchange market as a direct result of the capital inflow. Each of these is expressed as a fraction of the capital inflow. We have no fixed relationships among p, q and v, except that v must be equal to, or greater than, p. We must now postulate how capital entering Canada is spent, that is, the proportions spent on each of Penner's three types of uses. The proportions of each of these three types of capital inflows are part of the information we need to determine the values of p, q and v.

The easiest of the three types of capital inflows to estimate is the third; the capital spent on already existing real or financial assets in Canada. Penner has calculated that foreign purchases of outstanding stocks and bonds comprised 11.1 percent of the total net capital inflow between 1950 and 1959.[10] He also says that;

> In a private interview, a government official in a position to be informed on such matters estimated that at least 20 percent of direct foreign investment entering Canada during the late 1950's was used to finance purchases of established Canadian corporations.[11]

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[9] A capital inflow, in this analysis, will always mean a flow of capital into Canada.

[10] Penner, loc. cit., p. 532.

[11] Ibid., p. 532, n.9.

We will thus assume that the type 3 capital inflow represents 30 percent of the total. This, along with the value for p, will give us v. The proportions we should assign to capital inflows of types 1 and 2 is a much more indeterminant question. However, because Canada imports so much of the machinery and equipment she uses,[12] and because much of the investment in Canada by foreigners is in the resource industries where the capital equipment used is not of a type produced in Canada, we would expect that the proportion of the inflow used to buy imports would be relatively high. We will assume that 10 percent is a low estimate for capital inflows of type 2. This leaves us with 60 percent of the inflow of type 1. Relating these values to p, q and v, we have p = .6, v = .9, and q, as of yet, undetermined. The value of q will depend on whether the capital flow from country 2 (the United States), reduced aggregate demand in that country, and if so, by how much. Intuitively, we would think that the lending would not much affect home demand. We thus assume q = .1, realizing this to be the extreme upper limit that this parameter could assume under our other assumptions. We thus have the following set of parameters;

$m_{11} = 30$	$m'_{11} = .70$	$p = .6$
$m_{22} = 400$	$m'_{22} = .88$	$q = .1$
$m_{12} = 4$	$m'_{12} = .02$	$v = .9$
$m_{21} = 4$	$m'_{21} = .20$	

[12] Penner, loc. cit., p. 531, suggests that Canada imports over 40 percent of the machinery and equipment she uses.

with d_{11}^{+} , d_{22}^{+} , d_{12} and d_{21} still to be assigned values. Our two cross elasticities, as we saw before, are

$$d_{11}^{+} = \frac{m_{21}}{m_{11}} f_1 \; d_{21}$$

$$d_{22}^{+} = \frac{m_{12}}{m_{22}} f_2 \; d_{12}.$$

If we assign values to f_1 and f_2 , d_{11}^{+} and d_{22}^{+} will then be determined when values are assigned to d_{21} and d_{12}. Intuitively, we would expect the values of both f_1 and f_2 to be closer to 1 than to 0, and because of the relatively high proportion of Canada's imports that are machinery and equipment of a type not made in Canada, we would expect f_2 to be larger than f_1. Let us arbitrarily assume, then, that $f_2 = .9$ and that $f_1 = .7$. Substituting our parameter values into equations (3), (4) and (5), we have

$$A_1 = f_1 \, m_{21} \, d_{21} - d_{12} \, m_{12}$$

$$A_1 = -2.8d_{21} - 4d_{12}$$

and

$$A_2 = d_{21} \, m_{21} - f_2 \, m_{12} \, d_{12}$$

$$A_2 = 4d_{21} + 3.6d_{12}$$

and

$$A_3 = m_{21} + d_{12} \, m_{12} + d_{21} \, m_{21}$$

$$A_3 = 4 + 4d_{12} + 4d_{21}.$$

Setting $dY_1 = 0$, we have, after simplification

$$-.0120d_{21} + .1224d_{12} - .296 = 0. \quad \text{- - - - - - - - - - -(7)}$$

From equation (7)

if $d_{12} = 0$ then $d_{21} = -24.67$

$d_{12} = -1$ $d_{21} = -34.87$

$d_{12} = -2$ $\qquad$ $d_{21} = -45.07$

$d_{12} = -3$ $\qquad$ $d_{21} = -55.27.$

These are borderline conditions; that is, four sets of values for d_{12} and d_{21} which will make $dY_1 = 0$, given the assumed values for the other parameters. If, for any one of these four sets, d_{21} has a larger absolute value or d_{12} has a smaller absolute value, then dY_1 will be positive. These values of d_{21} are much larger than any estimates that have been calculated for the elasticity of demand for imports. Our best guess, then, is that if the values we have assumed for the other parameters are correct, dY_1 will be negative. We have, of course, no reason to expect that the parameters will have the values that we assumed them to have. We must, therefore, investigate some of the other possibilities.

The parameters for which we have the least information are d_{11}^{+} and d_{22}^{+}. We will now change these and see how our results are affected. If we assume that $f_1 = .5$, then $A_1 = -2d_{21} - 4d_{12}$. Setting $dY_1 = 0$, we have, after simplification

$$-.1d_{21} + .1224d_{12} - .296 = 0 \text{ - - - - - - - - - - - -} (8)$$

and thus

if $d_{12} = 0$ $\qquad$ then $d_{21} = -2.96$

$d_{12} = -1$ $\qquad$ $d_{21} = -4.18$

$d_{12} = -2$ $\qquad$ $d_{21} = -5.41$

$d_{12} = -3$ $\qquad$ $d_{21} = -6.63$

which is another set of borderline conditions. We notice, from examining equations (7) and (8), the great deal of

difference that changing f_1 from .7 to .5 has made; much more of a difference than changes in d_{12} make. If, along with our assumption that $f_1 = .5$, we assume that $f_2 = 1$, then $A_2 = 4d_{21} + 4d_{12}$, and we have

$$-.1d_{21} + .12d_{12} - .296 = 0 \quad \text{- - - - - - - - - - - - -(9)}$$

and thus

if		then	
	$d_{12} = 0$		$d_{21} = -2.96$
	$d_{12} = -1$		$d_{21} = -4.16$
	$d_{12} = -2$		$d_{21} = -5.36$
	$d_{12} = -3$		$d_{21} = -6.56.$

These values do not vary significantly from the ones derived from equation (8) and we can conclude, then, that the value of f_2 is not as critical to the sign of dY_1 as is the value of f_1.

In order to investigate how the propensities influence the sign of dY_1, it is convenient to regroup equation (1) into the form

$$dY_1 = \frac{da}{E}\Big\{ m'_{12}\ [p\ A_2 - q\ A_1 - q\ A_3 - v\ A_2] - (1 - m'_{22})[\ p\ A_3 + v\ A_1]. \text{- - - - -} \quad (10)$$

Now, assuming $f_1 = .5$, and $f_2 = 1$, and setting $dY_1 = 0$, we have, after simplification,

$$m'_{12}\ [-1.4d_{21} - 1.2d_{12} - .4] + (1 - m'_{22})\ [1.2d_{12} - .6d_{21} - 2.4] = 0. \text{- - -} \quad (11)$$

Now, if $m'_{12} = .03$ instead of .02, and if $m'_{22} = .88$, which means that the mps = .09, we have

$$-.114d_{21} + .108d_{12} - .300 = 0 \quad \text{- - - - - - - - - -} \quad (12)$$

from which we have the condition that;

if	$d_{12} = 0$	then	$d_{21} = -2.63$
	$d_{12} = -1$		$d_{21} = -3.57$
	$d_{12} = -2$		$d_{21} = -4.53$
	$d_{12} = -3$		$d_{21} = -5.47.$

Thus, increasing m'_{12} increases the chances of dY_1 being positive. If we assume that $m'_{22} = .9$ and that $m'_{12} = .02$, making the mps = .8, we have

$$.088d_{21} + .096d_{12} - .248 = 0 \text{ - - - - - - - - - - - - (13)}$$

thus

if	$d_{12} = 0$	then	$d_{21} = -2.82$
	$d_{12} = -1$		$d_{21} = -3.91$
	$d_{12} = -2$		$d_{21} = -5.00$
	$d_{12} = -3$		$d_{21} = -6.09.$

If we increase both m'_{22} and m'_{12} so that $m'_{22} = .9$, $m'_{12} = .03$ and the mps = .07, then we have

$$-.102d_{21} + .084d_{12} - .252 = 0 \text{ - - - - - - - - - - -(14)}$$

from which we can write that

if	$d_{12} = 0$	then	$d_{21} = -2.47$
	$d_{12} = -1$		$d_{21} = -3.29$
	$d_{12} = -2$		$d_{21} = -4.12$
	$d_{12} = -3$		$d_{21} = -4.94.$

If we apply these values of m'_{12} and m'_{22} to our original set of parameters, i.e., where $f_1 = .7$ and $f_2 = .9$, we have

$$-.0276d_{21} + .0876d_{12} - .252 = 0 \text{ - - - - - - - - - -(15)}$$

from which

if	$d_{12} = 0$	then	$d_{21} = -9.13$
	$d_{12} = -1$		$d_{21} = -12.30$

$d_{12} = -2$ $\qquad$ $d_{21} = -15.48$

$d_{12} = -3$ $\qquad$ $d_{21} = -18.65.$

These conditions vary significantly from the ones calculated using the smaller values of m'_{12} and m'_{22}.

We will now assume that the proportion of the capital inflow spent on already existing real and financial assets in Canada falls from 30 percent to 20 percent, and that there is a corresponding increase in spending on domestically produced goods of 10 percent. We now have $p = .7$, $q = .1$ and $v = .9$. For the original set of parameters, setting $dY_1 = 0$, equation (1) reduces to

$$-.0520d_{21} + .0816d_{12} - .344 = 0 \quad \text{- - - - - - - - - -(16)}$$

from which

if $d_{12} = 0$ $\qquad$ then $d_{21} = -6.62$

$d_{12} = -1$ $\qquad$ $d_{21} = -8.18$

$d_{12} = -2$ $\qquad$ $d_{21} = -9.75$

$d_{12} = -3$ $\qquad$ $d_{21} = -11.32.$

If we assume that $f_1 = .5$ and that $f_2 = 1$, then our equation reduces to

$$-.14d_{21} + .08d_{12} - .344 = 0 \quad \text{- - - - - - - - - - -(17)}$$

from which

if $d_{12} = 0$ $\qquad$ then $d_{21} = -2.46$

$d_{12} = -1$ $\qquad$ $d_{21} = -3.03$

$d_{12} = -2$ $\qquad$ $d_{21} = -3.60$

$d_{12} = -3$ $\qquad$ $d_{21} = -4.17.$

If we now assume that $m'_{12} = .03$ and that $m'_{21} = .9$, then equation (17) becomes

$$-.13d_{21} + .056d_{12} - .292 = 0 \quad \text{- - - - - - - - - - (18)}$$

from which

if	d_{12}	= 0	then	d_{21}	= -2.25
	d_{12}	= -1		d_{21}	= -2.68
	d_{12}	= -2		d_{21}	= -3.11
	d_{12}	= -3		d_{21}	= -3.54
	d_{12}	= -4		d_{21}	= -3.97.

There are, of course, an infinite number of other combinations of the elasticities and propensities which could be examined. Our analysis, however, has certainly reached the point of decreasing marginal returns, and further calculations would show, I think, nothing new. We have calculated in this section some of what we called the borderline conditions of our model for different assumed sets of parameters. To try to judge which of these sets of assumptions is most likely to be realized in the "real world" would require statistical estimations of all the parameters; a task which can certainly not be attempted here. Without such estimations, however, we can reach no definite conclusions, although this fact in itself is rather significant in view of the positive conclusions that have been reached by some economists, apparently without the aid of statistical estimates.

One other comment should be made in conclusion. The analysis in this section was definitely biased towards conditions which would make the change in national income positive. A set of parameters was assumed, and the borderline conditions found were seen to be rather restrictive. The parameter values were then "adjusted" so that

lower values of the elasticities would appear in the borderline conditions. If one knows that the elasticity of demand for imports must be in the neighborhood of -30 to insure a positive change in national income, it does not seem worthwhile to examine sets of conditions which would require even higher elasticities. However, the fact that we had to search for sets of parameters that would result in "normal" values for the elasticities in the borderline conditions, could be taken as an indication that the change in national income is most likely to be negative. Such a conclusion, however, rests on the assumption that the original assumed set of parameter values is a reasonably accurate reflection of the real world; an assumption which is not easily supported.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The object of this thesis was to examine the nature of the relationship which exists between international capital movements and the level of real national income and employment. After a brief review of the relevant literature, a simple, Keynesian, comparative statics model was set up, the assumptions of which were first of all examined in detail. The assumptions used were of a rather restrictive nature, but were deemed necessary in order to leave the model simple enough to handle. On completion of the mathematical formulation of the model, the stability conditions were discussed briefly, after which three specific types of capital inflows were examined, and the conditions which would make the change in income positive or negative were derived for each of the three types. An attempt was then made to show which of the parameters, when changed, most affected the level of national income. The final section was devoted to an examination of some specific borderline conditions of the general model in order that some idea could be gained as to what the effects of capital inflows might be for certain sets of assumed values of the parameters.

The conlusions of this thesis must be of a very general nature. We have seen that there are no a priori grounds for assuming that a capital inflow will have either a positive or negative effect on the level of real national income and employment. The change in national income, both the direction and size, will depend on the elasticities and propensities that exist in the two countries. Unfortunately, there do not seem to be any very reliable estimates of these parameters. For example, the cross elasticity of demand, and its major component, the substitution ratio (or the degree of substitution), have been, to this writer's knowledge, completely ignored. This is a rather serious matter when the importance of this parameter is considered. As our calculations showed, the value of d_{11}^{+} determines, to a larger degree than any other parameter, both the sign and magnitude of changes in real national income.

A number of minor conclusions, or observations, are worthy of mention. First of all, we found that the stability condition for our model, derived under the assumption that a positive determinant of the coefficient matrix insured stability, contains the Marshall-Lerner condition as a necessary, although not sufficient, condition for stability. The marginal propensities to save and to import, and the substitution ratios of the two countries were also found to be crucial factors in determining whether or not the model would be stable. We observed that

the sum of the elasticities of demand for imports of the two countries might need to be considerably greater than one, to insure the stability of the model. We realized, of course, that our Keynesian assumptions probably tended to distort our stability conditions, and we concluded that less restrictive assumptions about the money supply and the interest rate would probably lead to less stringent stability conditions.

Examination of Penner's type 1 capital inflows led to the conclusion that Penner's assumption of complete substitution is not compatible with his conclusion that national income will not be affected by a capital inflow, unless he is also prepared to assume that the elasticity of demand for imports is infinite. We also found that if a capital inflow is of type 1 or type 2, the sign of the change in income can be determined if the values of d_{21} and f_t are known. We found that country 2's elasticity of demand for imports could not affect the sign of the change in the income of country 1, if a capital inflow was of type 1 or type 2. This resulted from the fact that exchange rate changes and price changes were, by definition, the same thing. This, along with the interrelations of the exchange rate and the elasticity of demand for imports, was found to prevent d_{12} from influencing the sign of changes in the income of country 1.

Analysis of Penner's type 3 capital inflow did not lead to any such easily interpreted relationship. We were

able to show, however, by examining extremes, the conditions under which the change in national income would definitely be negative and those under which it could possibly by positive. We concluded that the circumstances which would result in a positive change in national income were rather unlikely from a theoretical point of view.

The investigation of the effects of the different parameters did not prove to be entirely satisfactory. The results depended so much on ititial conditions, which could be little more than random choices, that no exact conclusions could be reached. We did find that the propensities and elasticities of country 1 had more influence on the changes in dY_1 than the same parameters for country 2. We were also able to verify our conclusion from the section before, that country 2's elasticity of demand for imports could not affect the sign of change in country 1's income--this, of course, under the assumption that the capital inflow was of type 1. The elasticity d_{11}^{+} was also found to be a very influential parameter: even more important than country 1's elasticity of demand for imports.

The final section of Chapter III dealt with some specific cases of the general model, with the hope that such an analysis would show what we could expect the effect of capital inflows to be if certain sets of parameter values existed in the real world. The success of the analysis was limited by the uncertainty that surrounded all of the guesses as to what the parameter values would

be, and particularly by the fact that we had no idea at all of what values the most crucial parameters, the cross elasticities of demand, would take.

Throughout our analysis, we have constantly been aware that our model represents only a part of the "real world", and we realize that there are many forces which exist, but of which we cannot take account, and probably as many more of which we are not even aware. The ones which were considered important were discussed in Chapters I and II and a further listing of them here does not seem worthwhile. We must remember, however, that all these "other factors" would have to be analysed in detail before we could reach any definite conclusions as to the effects of capital inflows, even if we knew the values of the parameters of the model.

In summary, our conclusions are as follows. Capital inflows, from a theoretical point of view, can be either expansionary or contractionary. Before we can decide which will be the case, estimates of the parameters will have to be made. This is particularly important for the "substitution ratio" (f_1 and f_2), a crucial factor which has been almost completely ignored, and of which we know almost nothing. A complete analysis of all the factors not included in the model would also have to be made, for it may be that these other factors are more important in determining the change in dY_1 than the ones we have included in our model. Some attempt should be made to

determine their probable relative importance and the direction of their influence before definite conclusions are reached.

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APPENDIX A

SOHMEN'S ANALYSIS OF THE EFFECT OF EXCHANGE RATE CHANGES

Sohmen begins his analysis of the effects of exchange rate changes on employment by showing that an appreciation of a country's currency, which will lower the domestic prices of imports, will increase the country's real national product. He calls this the "real-income effect induced by the terms-of-trade change . . ."[1] He then goes on to say:

> If the values of exports and imports are forced into equality while their exchange ratio is altered, their physical volumes cannot have changed in the same proportion. They may, in fact, not even change in the same direction. A movement of the exchange rate, therefore, cannot fail to exert a direct influence on the size of the real national product, quite apart from the effect of changes in the terms of trade on domestic expenditure which we discussed above. . . . It will now be shown that the direct foreign-trade effect of appreciation of a country's currency as the demand for its exports increases must be depressive as long as domestic prices of each country's own output remain constant.[2]

To analyze the effects of this "increase in demand", Sohmen reasons, that, "all consequences of a rise in demand for exports can therefore be investigated by asking what happens as the currency appreciates while the values

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[1]Sohmen, op. cit., p. 98. [2]Ibid., p. 95.

of imports and exports remain in balance, . . ."[3]. Thus Sohmen is attempting to measure the effects of an increase in demand for exports indirectly by measuring the effects of a change in the exchange rate. Since capital inflows also appreciate the currency this procedure is equally valid for examining the effect of the inflow of foreign funds, for as Sohmen himself says:

> An autonomous inflow of funds, for example, increases the supply of foreign exchange exactly as if exports of goods and services had increased by the same amount.[4]

Sohmen then defines the following symbols:

M . . . . . physical quantity of imports

X . . . . . physical quantity of exports

r . . . . . exchange rate defined as units of domestic currency per unit of foreign currency

e . . . . . absolute value of the domestic elasticity of demand for imports.

Then, defining the commodity units in such a way that the domestic price of each country's exports becomes unity, and because of the continuous equality which must exist between the value of exports and imports, he can write

$$X = M \cdot r. \quad - (1)$$

Then differentiating (1) with respect to r:

$$\frac{dX}{dr} = M + \frac{dM}{dr} \cdot r \quad - - - - - - - - - - - - - - - - - - (2)$$

but $\frac{dM}{dr}$ can be written as

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[3]Ibid., p. 96. [4]Ibid., p. 19.

$$\frac{dM}{dr} = \frac{dM}{dr} \cdot \frac{r}{M} \cdot \frac{M}{r}$$

$$\frac{dM}{dr} = -e \cdot \frac{M}{r} \text{ - } (3)$$

Substituting (3) into (2) he gets

$$\frac{dX}{dr} = M - e \frac{M}{r} \cdot r$$

$$\frac{dX}{dr} = M (1 - e) \text{ - - - - - - - - - - - - - - - - - - } (4)$$

Sohmen then takes the ratio

$$\frac{dX/dr}{dM/dr} = \frac{M(1 - e)}{-e \cdot M/r}$$

$$\frac{M(1 - e)r}{-eM}$$

$$\frac{dX/dr}{dM/dr} = r(1 - \frac{1}{e}) \text{ - - - - - - - - - - - - - - - - - } (5)$$

Then by redefining the unit of foreign currency so as to make r = 1 in the initial equilibrium and by assuming that "changes in the volumes of exports and imports by the same absolute amount have effects of equal magnitude, but of opposite sign, on real national product and employment"[5], he writes

$$R = 1 - \frac{1}{e} \text{ - } (6)$$

where R presumably is equal to $\frac{dX/dr}{dM/dr}$, and somehow represents the change in national income and employment. He then goes on to say:

> In the absence of Giffen's paradox, the denominator of (5) is always negative. A positive value for R therefore indicates that the volume of exports will rise after an increase in foreign demand.[6]

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[5]Ibid., p. 97. [6]Ibid.

This conclusion of Sohmen's does not make any sense, nor for that matter does equation (6), for what Sohmen is saying, is that when the exchange rate appreciates, which will lower the domestic price of our imports, and, of course, increase the price which foreign countries must pay for our exports, both imports and exports will increase. At the same time, he assumes the "absence of Giffen's paradox". These two are clearly incompatible, for the "absence of Giffen's paradox" means that when prices rise the quantity bought will decrease, and vice versa.[7] Thus, when the currency appreciates, our imports must increase and our exports must decrease, which means that R must be negative. From Sohmen's equation (5), this is impossible if the elasticity of demand for imports is greater than, or equal to, one.

This obvious contradiction results from Sohmen's failure to specify the sign of the derivatives in the development of his model, and from his apparent belief that somehow or other he has included in his model the effects of an increase in foreign demand. We will examine each of these in turn.

In his analysis, i.e., equations (1) to (6), Sohmen has apparently assumed that both derivatives, i.e., both dX/dr and dM/dr, have the same sign. This is apparent

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[7] This is not, strictly speaking, correct, for if prices rise and the quantity bought remains constant, Giffen's paradox is still absent. We will assume that this does not occur.

from equation (3) he assumes that dM/dr is negative and thus he also assumes that dX/dr is negative, as is evident from equation (4) (as long as e > 1). This, of course, is contrary to his assumption that Giffen's paradox is absent. Sohmen seems to be trying to reconcile his mistaken impression that dX and dM have the same sign by assuming that somehow the increase in exports resulting from an increase in foreign demand has been included in his analysis. While it is certainly true that an increase in foreign demand for a country's exports would appreciate the value of the currency, this increase in exports is certainly not included in the above analysis; it is something entirely separate which must be considered along with the changes in imports and exports brought about by the exchange rate change, in determining the total effect of an increase in foreign demand.

Had Sohmen carried out his analysis correctly, he would have arrived at the expression:

$$\frac{dX/dr}{dM/dr} = -(1 + \frac{1}{e})$$

where all terms are defined as before. The left-hand expression of (7) must always be negative, which means that exports and imports will always move in opposite directions when the exchange rate is changed. The effect of these changes on national income is still indeterminant, however, for (7) only gives us the value of the ratio of the change in exports to the change in imports, and we must know the exact magnitude of each of these components to determine what the effect on national income would be.

The only conclusions we can reach, then, is that imports and exports will move in opposite directions when the exchange rate changes. This is the same thing as the "absence of Giffen's paradox" tells us. Thus the analysis is not seen to be very significant.

Sohmen next attempts to show that after a decrease in the exchange rate, the depressive foreign-trade effect will always be greater than the positive real-income effect. He is attempting to prove, in other words, that when the exchange rate decreases, the net effect of this decrease will be a reduction in real national income. Having shown this to his satisfaction he then concludes that:

> Strictly Keynesian assumptions leave us, indeed, with the result that under flexible exchanges and in the absence of captial transfers a boom in one country will lower national income and employment in the rest of the world.[8]

Sohmen, however, has still failed to include the direct effect of this boom. His analysis has dealt only with the indirect effects of the increase in foreign demand, i.e., the effects of the exchange rate change. His conclusion, then, cannot be accepted, for while what he says may certainly be so, he has failed to prove it.

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[8] Ibid., p. 99.

APPENDIX B

CONDITIONS FOR STABILITY OF THE MODEL

In Chapter II it was shown that the necessary and sufficient condition for stability of the model was that the determinant of the coefficient matrix be positive. This determinant was expressed as

$$E = A_1[m'_{12}\,m'_{21} - m'_{21}(1 - m'_{22})] - A_2[m'_{21}\,m'_{12} - m'_{12}(1 - m'_{11})] - A_3[(1 - m'_{22})(1 - m'_{11}) - m'_{21}m'_{12}].$$

As it stands this expression is impossible to evaluate in any general sense. A simplification of it is therefore desirable. To begin with, we will set $E = 0$ (because zero is the borderline between stability and instability), and we will assume that the values of the propensities for the two countries are the same. We can then write

$$A_1[m'_{21}\,m'_{21} - m'_{21}(1 - m'_{11})] - A_2[m'_{21}\,m'_{21} - m'_{21}(1 - m'_{11})] - A_3[(1 - m'_{11})(1 - m'_{11}) - m'_{21}m'_{21}] = 0$$

$$[m'_{21}\,m'_{21} - m'_{21}(1 - m'_{11})][A_1 - A_2] - A_3[(1 - m'_{11})(1 - m'_{11}) - m'_{21}m'_{21}] = 0$$

$$m'_{21}\,m'_{21}[A_1 - A_2] - m'_{21}(1 - m'_{11})[A_1 - A_3] - (1 - m'_{11})(1 - m'_{11})\,A_3 + m'_{21}\,m'_{21}\,A_3 = 0$$

$$m'_{21}\,m'_{21}[A_1 - A_2 + A_3] - m'_{21}(1 - m'_{11})[A_1 - A_2] - (1 - m'_{11})(1 - m'_{11})\,A_3 = 0$$

$$m'_{21}m'_{21}[A_1 - A_2 + A_3] - (1 - m'_{11})\{m'_{21}[A_1 - A_2] + (1 - m'_{11})A_3\} = 0$$

$$m'_{21}m'_{21}[A_1 - A_2 + A_3] - (1 - m'_{11})[m'_{21}A_1 - m'_{21}A_2 + (1 - m'_{11})A_3] = 0.$$

Now, because $m'_{11} + m'_{21}$ + mps (the marginal propensity to save) must add to unity, we can write $(1 - m'_{11})$ as m'_{21} + mps, or, since the mps can be written as $(1 - m'_{11} - m'_{21})$, we have

$$(1 - m'_{11}) = m'_{21} + (1 - m'_{11} - m'_{21}).$$

Thus,

$$m'_{21}m'_{21}[A_1 - A_2 + A_3] - (1 - m'_{11})[m'_{21}A_1 - m'_{21}A_2 + m'_{21}A_3 + (1 - m'_{11} - m'_{21})A_3] = 0$$

$$m'_{21}m'_{21}[A_1 - A_2 + A_3] - (1 - m'_{11})[m'_{21}A_1 - m'_{21}A_2 + m'_{21}A_3] - (1 - m'_{11})(1 - m'_{11} - m'_{21})A_3 = 0$$

$$m'_{21}m'_{21}[A_1 - A_2 + A_3] - (1 - m'_{11})m'_{21}[A_1 - A_2 + A_3] - (1 - m'_{11})(1 - m'_{11} - m'_{21})A_3 = 0$$

$$m'_{21}[A_1 - A_2 + A_3][m'_{21} - (1 - m'_{11})] - (1 - m'_{11})(1 - m'_{11} - m'_{21})A_3 = 0$$

$$m'_{21}[A_1 - A_2 + A_3](m'_{21} - 1 + m'_{11}) - (1 - m'_{11})(1 - m'_{11} - m'_{21})A_3 = 0$$

$$-m'_{21}[A_1 - A_2 + A_3](1 - m'_{11} - m'_{21}) - (1 - m'_{11})(1 - m'_{11} - m'_{21})A_3 = 0$$

$$-m'_{21}[A_1 - A_2 + A_3] - (1 - m'_{11})A_3 = 0.$$

Now

$$A_1 = d^+_{11}m_{11} - d_{12}m_{12}$$

$$A_2 = d_{21}m_{21} - d^+_{22}m_{22}$$

$$A_3 = m_{21} + d_{12}m_{12} + d_{21}m_{21}$$

Therefore

$$A_1 - A_2 + A_3 = d_{11}^{+} m_{11} - d_{12} m_{12} - d_{21} m_{21} + d_{22}^{+} m_{22}$$
$$+ m_{21} + d_{12} m_{12} + d_{21} m_{21}$$
$$= d_{11}^{+} m_{11} + d_{22}^{+} m_{22} + m_{21}.$$

Then

$$- m'_{21} [m_{21} + d_{11}^{+} m_{11} + d_{22}^{+} m_{22}] - (1 - m'_{11})$$
$$[m_{21} + d_{12} m_{12} + d_{21} m_{21}] = 0.$$

We know, however, that

$$d_{11}^{+} = \frac{\partial m_{11}}{\partial r} \frac{r}{m_{11}}, \text{ and } d_{21} = \frac{\partial m_{21}}{\partial r} \frac{r}{m_{21}}$$

or

$$r = \frac{\partial r}{\partial m_{11}} m_{11} d_{12}, \text{ and } r = \frac{\partial r}{\partial m_{21}} m_{21} d_{21}$$

therefore

$$d_{11}^{+} = \frac{\partial m_{11}}{\partial r} \frac{\partial r}{\partial m_{21}} \frac{m_{21}}{m_{11}} d_{21}$$

or

$$d_{11}^{+} = f_1 \frac{m_{21}}{m_{11}} d_{21}$$

where

$$f_1 = \frac{\partial m_{11}}{\partial r} \frac{\partial r}{\partial m_{21}}$$

similarly

$$d_{22}^{+} = f_2 \frac{m_{12}}{m_{22}} d_{12}$$

where

$$f_2 = \frac{\partial m_{22}}{\partial r} \frac{\partial r}{\partial m_{12}}.$$

We can now write

$$- m'_{21} [m_{21} + m_{21} f_1 d_{21} + m_{12} f_2 d_{12}]$$
$$- (1 - m'_{11})[m_{21} + d_{12} m_{12} + d_{21} m_{21}] = 0.$$

Now assuming $m_{21} = m_{12}$

$$- m'_{21} m_{21} [1 + f_1 d_{21} + f_2 d_{12}] - (1 - m'_{11}) m_{21}$$
$$[1 + d_{12} + d_{21}] = 0$$

or

$$- m'_{21} [1 + f_1 d_{21} + f_2 d_{12}] - [m'_{21} + (1 - m'_{21} - m'_{11})]$$
$$[1 + d_{21} + d_{12}] = 0$$

$$- [1 + f_1 d_{21} + f_2 d_{12}] - \left[1 + \frac{(1 - m'_{21} - m'_{11})}{m'_{21}}\right]$$
$$[1 + d_{21} + d_{12}] = 0$$

$$\left[1 + \frac{(1 - m'_{21} - m'_{11})}{m'_{21}}\right][- 1 - d_{21} - d_{12}]$$
$$- [1 + f_1 d_{21} + f_2 d_{12}] = 0.$$

Changing to absolute values we have

$$\left[1 + \frac{(1 - m'_{21} - m'_{11})}{m'_{21}}\right] [-1 + d_{21} + d_{12}]$$
$$- [1 + f_1 d_{21} + f_2 d_{12}] = 0$$

$$- 1 + d_{21} + d_{12} + \frac{(1 - m'_{21} - m'_{11})}{m'_{21}} [-1 + d_{21} + d_{12}]$$
$$- 1 - f_1 d_{21} - f_2 d_{12} = 0$$

collecting terms

$$d_{21} - f_1 d_{21} + d_{12} - f_2 d_{12} - 2 + \frac{(1 - m'_{21} - m'_{11})}{m'_{21}}$$
$$[(d_{21} + d_{12}) - 1] = 0$$

$$\frac{(1 - m'_{21} - m'_{11})}{m'_{21}} [(d_{21} + d_{12}) - 1] + d_{21} (1 - f_1)$$
$$+ d_{12} (1 - f_2) - 2 = 0.$$

Thus the model is stable if

$$\frac{(1 - m'_{21} - m'_{11})}{m'_{21}} [(d_{21} + d_{12}) - 1] + d_{21}(1 - f_1) + d_{12}(1 - f_2) > 2 - - - - (1)$$

and if $f_1 = f_2 = 1$, the stability condition becomes

$$\frac{(1 - m'_{21} - m'_{11})}{2m'_{21}} [(d_{21} + d_{12}) - 1] > 1 - - - - - (2)$$

which are the two equations we examined in Chapter II.

Equation (1) can be converted into an expression for E by bringing back into the equation terms which we cancelled out. Thus,

$$E = [(1 - m'_{11} - m'_{12})(m'_{12})(m_{21})] \left\{ \frac{(1 - m'_{21} - m'_{11})}{m'_{21}} [(d_{21} + d_{12}) - 1] + d_{21}(1 - f_1) + d_{21}(1 - f_1) - 2 \right\}$$

is an alternative expression for E.